DOCUMENT RESUME

ED 237 354	SE 043 367

TITLE	Mathematics. High School Curriculum Objectives.
INSTITUTION	Boston Public Schools, Mass.
PUB DATE	83
NOTE	246p.
AVAILABLE FROM	Instructional Services, Boston Public Schools, 26 Court Street, Boston, MA 02108.
PUB TYPE	Guides - Non-Classroom Use (055)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Algebra; *Calculus; *Educational Objectives; *Geometry; High Schools; Mathematics; Mathematics Curriculum; Mathematics Education; *Secondary School Mathematics; *Trigonometry
IDENTIFIERS	*Massachusetts (Boston)

ABSTRACT

This document lists mathematics objectives for Boston high school students. All objectives are presented in two columns. The left-hand column states each objective in general terms and gives an idea of its scope. The right-hand column gives a specific example of what students should be able to do when the objective is achieved. Objectives are provided for: (1) basic mathematics/level six; (2) general mathematics/level seven; (3) pre-algebra (algebra I prerequisites); (4) algebra I; (5) geometry; (6) algebra II; (7) trigonometry; (8) advanced mathematics; and (9) differential and integral calculus. Each part begins with introductory comments and includes a list of the areas/sections into which the objectives are divided. For example, objectives for basic mathematics/level six (designed for students who enter high school without having passed the Massachusetts Basic Skills test in mathematics) are divided into: numbers/numeration; computation; fractions; decimals/percents; measurement; geometry; estimating, graphs, and other areas; and basic mathematics and careers. The document also includes an introductory section (discussing how objectives were developed, course sequence, placing students in sequence, issues raised by the objectives, and others areas) and a list of 16 goals. (JN)

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MATHEMATICS

HIGH SCHOOL CURRICULUM OBJECTIVES

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MESSAGE FROM THE SUPERINTENDENT

This set of curriculum objectives for high school Mathematics builds on the strong foundation of our Elementary and Middle School Mathematics document. The high school objectives continue the K-8 philosophy, stressing basic mathematics skills as well as problem-solving applications, and bringing students from fundamental mathematics to advanced placement Calculus.

These objectives are comprehensive and demanding, conveying high expectations of our students. They pose a challenge to all those who teach Mathematics in our high schools, and also those in our elementary and middle schools, to lay the groundwork for independent adult learning. We must look upon the job of increasing our students' mathematical power as a long-term endeavor in which we are all part of a single team. Our challenge is to create a cohesive, cooperative effort in each school, so that regular education, special education, bilingual, advanced work, and funded-program teachers work together with administrators, parents, university personnel, school volunteers, and others to improve the achievement of all students.

While this document states clear curriculum objectives for students at each grade level, it allows considerable flexibility for teachers, department heads, and headmasters to determine the most effective methods and materials. There must be common citywide goals and objectives, but those who work most closely with students are in the best position to decide how to convey the curriculum. Thus the purpose of this document is to harness the efforts of all school personnel in a common endeavor, and at the same time liberate the creative energy and dedication in each of our schools.

I want to thank all those who contributed to this document. It is a remarkable synthesis of previous work from inside and outside the Boston schools, and has benefited from generous amounts of feedback from numerous individuals. It is this breadth of source material and input that makes the objectives so strong, authoritative, and potentially helpful in our classrooms.

Robert R. Spillane
Superintendent of Schools

CONTENTS

Introduction............................i

Philosophy and Goals....................x

Basic Mathematics/Level Six.............1

General Mathematics/Level Seven........19

Pre-Algebra (Algebra I prerequisites)..37

Algebra I..............................44

Geometry...............................65

Algebra II............................109

Trigonometry..........................133

Advanced Mathematics..................141

Differential and Integral Calculus....182

Acknowledgements......................230

INTRODUCTION

The purpose of this document is to present clear objectives in mathematics for all Boston high schools. The objectives are a statement of what students should have mastered by the end of each course.

These objectives continue the sequence begun by the Elementary and Middle School Mathematics Curriculum Objectives approved in 1982. The K-8 objectives cover the mathematical steps prior to algebra; the high school objectives cover the overlapping areas of Basic and General Mathematics and then complete the full college preparatory and advanced placement sequence. Together, the K-8 and 9-12 objectives will form a complete curriculum in mathematics from kindergarten through high school graduation.

This document is intended to clarify what students should know; it does not deal with how students should be taught. Those who work directly with students know best how to resolve the many instructional questions raised by these objectives. Therefore, decisions on methods and materials remain in the hands of teachers and department heads, with help when needed from headmasters, central office support personnel, university consultants, and others.

How the objectives were developed

Many others inside and outside the Boston schools have developed curriculum objectives in high school mathematics. Rather than reinventing the wheel, we drew heavily on previous work. Among the resources were:

- previous Boston curriculum documents
- objectives, scope and sequence documents, and final exams from Boston high schools
- exemplary curriculum documents from other school systems around the country
- a sampling of recent textbooks and instructional materials in high school mathematics

Three consultants (see Acknowledgements) compiled the original draft objectives, drawing on these resources and using the two-column format that is a feature of all our curriculum documents. In June, 1983, copies of the draft objectives were circulated to all Boston high school mathematics teachers, to more than a hundred outside experts, and to interested parents. All were invited to critique the curriculum, and more than three hundred separate response sheets were received. They reflected strong support for the objectives, and included many specific comments and criticisms.

Consultants worked on incorporating the many excellent suggestions made by readers, and on insuring smooth transitions from one set of objectives to another. We feel that the finished product represents a consensus of Boston teachers' judgments about what a high school curriculum should contain, as well as the wisdom of a number of outside mathematics experts.

The two-column format

All objectives are presented in two columns. The left-hand column states each objective in general terms and indicates its scope. The right-hand column gives a specific example of what students should be able to do when the objective is achieved. The right-hand column can serve as a model for teacher-made tests and will be the blueprint for eventual citywide competency examinations.

Why the objectives were written

The new K-12 curriculum objectives and the policy that accompanies them were written in response to two related concerns in the Boston school community: first, that there was no citywide consistency in mathematics curriculum that linked objectives from classroom to classroom, grade to grade, and school to school; and second, that students' mathematical achievement was not as high as it needs to be for the technological age in which students will live and work. The current national interest in mathematics and science education strongly reinforces this concern.

Our strategy for addressing this challenge has five components, not all of which are in place at this point:

1. Citywide curriculum objectives for each grade level and course;
2. A uniform course sequence for all students;
3. A standard procedure for placing students at the right level in this progression;
4. Citywide examinations geared to the objectives for each level and course;
5. A gradual increase in the mathematics requirement for high school graduation.

The material below addresses each of these five areas, and then takes up several other issues touching on the mathematics curriculum for high school.

1. Curriculum objectives

With the publication of this document, no teacher will be without clear and authoritative objectives for mathematics courses from K-12. The material in each course is geared to what is generally expected in schools across the country, and reflects what is covered in textbooks and other instructional materials. The objectives for Basic and General Mathematics are drawn directly from Level Six and Seven of the K-8 curriculum objectives document published in 1982 (see below for a fuller discussion of this issue).

The objectives for each course are both a maximum and a minimum curriculum. The total list of objectives is ambitious, reflecting very high expectations of our students. Nested within this maximum curriculum are a set of minimum essentials. These priority objectives, which are circled, should be mastered by all students taking each course.

2. Sequence of courses

The curriculum objectives K-12 are mapped out in spiral fashion, with gradual increments in the level of difficulty and the amount of new content at each level. This makes it very important that students move through the objectives in a particular sequence - although the speed at which they move will vary from student to student and school to school. Below is the full K-12 sequence, stated in terms of levels:

ELEMENTARY & MIDDLE SCHOOL	**HIGH SCHOOL**
Kindergarten	
Level One	
Level Two	
Level Three	
Level Four	
Level Five	
Level Six - - - - - - - -	Basic Mathematics
Level Seven - - - - - -	General Mathematics
Level Eight - - - - - -	(Consumer Mathematics - an elective)
	Algebra I
	Geometry
	Algebra II (including Trigonometry)
	Advanced Mathematics
	Calculus

Basic Mathematics covers the objectives in Level Six of the K-8 sequence; General Mathematics covers Level Seven objectives. Students who pass General Mathematics should be prepared for Algebra I. (A list of pre-algebra skills is presented just before the Algebra I objectives; these are not intended to be a separate course.) Consumer Mathematics, an elective for students who have passed General Mathematics but do not want to embark on the algebra-geometry sequence, is based on the more applied Level Eight objectives.

3. Placing students in the sequence

There are currently hundreds of students in our high schools with low mathematics achievement. Educational research and common sense tell us that these students should be working at the level in the K-12 sequence at which they can experience success and be appropriately challenged. Placement of students at the right point in the sequence is therefore of critical importance.

In elementary and middle schools, students should be promoted from a given level when they have passed the level's competency test. For students entering high school, and for those in high school whose level is unknown, a citywide placement test is necessary. All entering ninth-grade students (and late arrivals in other grades) will take a placement test (developed by the Testing Office) designed to assess their level in the K-12 sequence. Predetermined cut-off scores will enable each high school mathematics department to divide students into four groups:

1. The lowest group includes all those who failed the State Basic Skills Test. This group takes Basic Mathematics, focusing on the Level Six objectives from the K-8 sequence. The purpose of this course is to help students re-take the State Basic Skills Test until they pass, and are prepared to move into General Mathematics.

2. The second group takes General Mathematics, focusing on the Level

Seven objectives from the K-8 sequence. Some of these students may move on to Algebra I after passing the General Mathematics competency test; others may take electives outside the college sequence.

3. The third group embarks on the college sequence with Algebra I.

4. The highest group has already taken Algebra I in eighth grade and passed the competency exam, and is placed in Geometry.

Below are possible course sequences for students at the four levels. Note that the length of time spent in each course could vary from school to school, changing the sequence somewhat.

	BASIC	**GENERAL**	**COLLEGE**	**ACCELERATED**
9th Grade	Basic Math	General Math	Algebra I	Geometry
10th Grade	General Math	Elec./Alg. I	Geometry	Algebra II
11th Grade	(Elec./Alg.I)	(Elec./Geometry)	Algebra II	Advanced Math
12th Grade	(Elec.)	(Elec.)	Advanced Math	Calculus

Any electives in addition to these courses (such as Consumer Mathematics, Applied Mathematics, Probability and Statistics, Vocational Mathematics, etc.) are offered at the discretion of each high school's mathematics department. However, students must have achieved at least the level of the end of General Mathematics/Level Seven before moving on to elective courses. Citywide objectives and competency exams may be developed for additional areas in the future.

Looking ahead to the next few years, we can anticipate a gradual change in the percentages of students entering various high school courses. As the more rigorous K-8 Mathematics objectives take hold and the proposed promotional policy is implemented, the mathematical level of entering ninth-grade students should steadily improve. This should decrease the number of high school students in Basic and General courses, and increase the percentage of students working in the college sequence and in electives that strengthen marketable skills.

4. Competency tests

A critical part of boosting students' achievement is insuring that they come into each course with the prerequisite skills and knowledge. This is where citywide competency tests at the end of each semester and year can play such an important role. Written in the central testing office in consultation with the Mathematics Council, these exams will be directly based on the objectives for each course, so that teachers and students know exactly what will be tested (without, of course, knowing the precise content of each test). These citywide competency tests will replace individual schools' final exams.

Each set of objectives in the final document will be broken into two sections. The first should be covered in the first semester, followed by a citywide mid-year exam; the second should be covered in the second semester, followed by the citywide final exam. Students who fail at mid-year will need some form of remedial instruction, or, if it is feasible, should re-take the first-semester course. Although this division of each set of curriculum objectives into semesters somewhat restricts teachers' flexibility, there are still options within each semester's work.

5. Increased high school mathematics requirements

The Mathematics Council has proposed the following high school graduation requirements in mathematics, beginning with students who entered ninth grade in 1982:

1. All students must take and pass two full years of mathematics.
2. General Business, Bookkeeping, Accounting, and Record Keeping courses cannot be counted toward this two-year mathematics requirement.

The Mathematics Council will also propose that, beginning with students who enter ninth grade in 1984, passing General Mathematics/Level Seven should be a high school graduation requirement. A further increase in the course requirement may be proposed for the 1985-6 school year.

These proposed requirements are more rigorous than those prior to 1983 in the following ways: first, they are geared to comprehensive citywide curriculum objectives, with students' performance measured by citywide competency exams correlated to those objectives; second, the elimination of General Business, Bookkeeping, Accounting, and Record Keeping courses as a partial means of fulfilling the mathematics requirement would increase the number of "pure" mathematics courses taken by students in many high schools; and third, the General Mathematics achievement level is higher than existing standards.

A further rationale for moving away from counting General Business, Bookkeeping, Accounting, and Record Keeping toward the mathematics requirement is that it would boost the level of mathematical preparation of students taking more technical courses in their home high schools and at the Humphrey Occupational Resource Center.

Both proposals are under consideration, and comments are welcomed.

OTHER CONCERNS

There are several other issues raised by the new high school curriculum objectives. These concerns are presented below in question and answer form for easy reference.

Which curriculum decisions remain in teachers' hands?

The objectives in this document clarify what Boston students should be able to do at the end of each mathematics course, and citywide competency tests will help focus attention on the objectives and give teachers useful feedback on students' progress and needs. But the most important decisions - those that really determine how much students learn - cannot and should not be made by a central office document. Below is a list of some of the decisions that teachers (supported by department heads, the Institute for Professional Development, the Senior Program Director for Mathematics, and others) should make:

- Readiness and review - giving students the foundations they need to move on to more advanced work
- Instructional sequence - deciding on the best order to attack objectives (although a sequence is suggested in the objectives, and certain objectives should be covered in the first semester)
- Blending - finding ways to mix objectives from different parts of the curriculum in innovative ways
- Materials - seeking out the most effective textbook material, workbooks, worksheets, overhead transparencies, manipulatives, and class activities to convey the objectives to students (the Director of Mathematics will be of assistance in suggesting materials which the best match with the citywide objectives)
- Enrichment - deciding how many of the uncircled objectives to include, and considering other extensions to the objectives
- Relevance - finding ways to make mathematics applicable to students' everyday lives
- Problem-solving - helping students develop step-by-step, imaginative approaches to attacking problems of all kinds
- Calculators and computers - deciding on the best applications for electronic aids in the curriculum, and helping students master them

Why aren't the high school objectives for Basic and General Mathematics distinct from the Level Six and Seven objectives?

Although methods and materials for teaching low- and middle-level high school students should be different from those used with middle school students, the basic objectives are the same. Therefore, a distinct set of high school objectives in Basic and General Mathematics would have duplicated the upper levels of the K-8 document.

Continuity from middle school to high school is crucial. Using the same objectives for students entering high school below Algebra I level will insure a smooth progression for students moving from middle to high school.

How are Basic and General Mathematics different from Level Six and Level Seven courses?

While working toward the same curriculum objectives as the upper levels of the K-8 document, Basic and General courses emphasize practical applications, and make optimal use of calculators, computer-assisted instruction, and age-appropriate materials that will engage students' interest and give them successful experiences with mathematics. These courses may also include material above and beyond what is specified in the objectives. The remedial high school courses leave the way open for students to join the college-preparatory curriculum if they reach that level of achievement and so desire.

Finding the best methods and materials for remedial students in high school is the main challenge for teachers of Basic and General courses. Many of these students have experienced a degree of frustration and failure in mathematics in elementary and middle school. This is especially true for the lowest-level students who have failed the State Basic Skills Test. Traditional approaches have not worked for them, and they have fallen further and further behind their more mathematically-oriented peers. A re-hash of previous approaches and materials is seldom successful in breaking the pattern of failure. The challenge is to find ways of teaching these students basic mathematics so that they become competent, more successful, and emerge from high school with the mathematical skills and literacy necessary for their adult lives.

Are Basic and General Mathematics really high school courses?

Using objectives from the K-8 sequence in high school mathematics courses raises an issue that has always existed in remedial high school courses: are they really high school courses? What should students and parents be told about the grade level of the work? The best response to this question is:

a). Students who are below grade level must be taken at their real level of achievement and accelerated through a sound developmental sequence; if their real achievement level is Level Six in mathematics, then that is where the remedial process must begin.

b). The Levels Six and Seven objectives are taught differently in high school Basic and General courses from the way they are in middle schools. The material used is appropriate to high school students and the methods and teaching styles are adapted to the needs of this special population.

Thus, Basic and General Mathematics, although based on lower-level curriculum objectives, are indeed high school courses.

Can students take courses that are too easy or too difficult for them?

The goal of a set of sequential curriculum objectives, citywide competency examinations, and course requirements is to help teachers place students at the level at which they are appropriately challenged. Students should take courses that are neither too easy nor too difficult for them.

In the past, there have been deviations from optimal student placement at both ends of the spectrum. Some students, whether from an unrealistic sense of their mathematical prowess or because of parental pressure, have elected to take courses for which they were unprepared - and the results have been unfortunate. Other students, although able to take more difficult courses, have elected to take lower-level courses to avoid algebra and geometry. In so doing, they have taken valuable instructional attention away from the truly needy, and have denied themselves the mathematical instruction from which they could benefit and to which they are entitled.

Both types of inappropriate placement have caused problems for teachers and students. We hope that both will be remedied by the policy set forth in this introduction. Students should not be allowed to enroll in courses for which they do not have the prerequisites (as measured by citywide competency examinations and placement tests); and, unless a student deliberately performs poorly on a placement test, no student should end up taking a course significantly below his or her current level of achievement. (Naturally, test results will have to be supplemented by individual counseling by mathematics teachers and high school counselors, taking into account other non-academic variables such as attendance, motivation, etc.)

Is a different sequence and pace possible in algebra and geometry?

Mathematics educators generally agree that the separation of algebra from geometry in high school courses is not ideal; there is considerable support for a more integrated curriculum such as that used in Europe and a few American public and private schools. However, this opinion is accompanied by skepticism that the traditional algebra-geometry separation reflected in most textbooks and school curriculum guides can be unseated.

While the objectives in this document follow the conventional model in separating algebra from geometry, the intent is not to lock all teachers into this approach. One advantage of having citywide objectives spelled out in a single document is that a mixing and matching of objectives from different courses is possible. Thus it will be possible to experiment with a more integrated curriculum in the years to come, as Dorchester High School has done in recent years. But to avoid a re-fragmentation of high school mathematics curriculum, any variations from the sequence and course division laid out in this document must be approved in advance by the Director of Mathematics.

At what level should calculators be used in coursework and tests?

Mathematics educators across the country are debating the appropriate level at which students should be encouraged to use calculators in the classroom and allowed to use them in testing situations. There seems to be little debate that upper-level students who have mastered basic mathematics should use calculators as a way of extending their problem-solving power in algebra, geometry, and more advanced courses. But for students below that level, a debate continues between those who want students to spend more time doing pencil-and-paper computing and those who argue that students will become increasingly disenchanted with mathematics - and unwilling to apply it in their lives - if they are not taught to use calculators as early as possible.

Clearly a cut-off point has to be established, and the following compromise policy has been approved by the Mathematics Council: students who have passed the State Basic Skills Test at the eighth-grade level will be encouraged to use calculators in general mathematics classes and where appropriate in higher courses. Thus, after mastering the basics of computation and passing a basic competency test (roughly equivalent to Level Six in the K-8 document), students should begin to rely on calculators to strengthen their mathematical powers and increase their success in mathematics courses. Teachers in elementary and middle schools may also find ways of using calculators to strengthen students' problem-solving power and ability to deal with cumbersome computation.

What is the relationship between mathematics objectives and computer literacy?

There has been a tendency to see computer education as part of the mathematics curriculum - it is associated with mathematical reasoning and high technology, and much of the available computer-assisted instruction (CAI) software contains drill and practice in mathematics. But, as argued in the introduction to the K-8 document, it seems clear that computer objectives should be both broader than and separate from the mathematics curriculum. Both the K-12 computer literacy and programming objectives and the high school computer literacy graduation requirement will take shape apart from the mathematics objectives documents.

To be sure, there will be an emphasis on computer materials and applications in the follow-up materials developed to help implement these objectives, and some mathematics teachers may be involved in teaching computer education courses. But the core mathematics curriculum objectives concentrate on the more traditional skills and knowledge.

What follow-up will there be to the publication of objectives?

Citywide curriculum objectives are only the first step in the overall curriculum strategy. They are being followed up by:

- tests that replace school-by-school semester and final exams and give teachers feedback on how their students are performing
- a systematic evaluation of textbooks and other instructional materials to find those that most effectively convey our curriculum objectives;
- opportunities for teachers to share effective materials and practices;
- support and training of teachers in areas where help is needed, in the form of workshops, courses, and in-service sharing;
- regular professional updates on developments in the field of mathematics by the newly appointed Senior Program Director for Mathematics.

A final word

The curriculum policy described in the pages above strikes a balance between systemwide uniformity and school-by-school, classroom-by-classroom diversity. The curriculum objectives, sequence of instruction, competency examinations, and prerequisites for each level and course are mandatory. The way the objectives are taught - including the length of courses (e.g., a one-, two-, or three-semester Algebra I course), the methods and materials used (textbooks, worksheets, audio-visuals, computer-assisted instruction, etc.), and the electives offered to extend and enrich the core curriculum - are left to individual schools. The object is to set clear and consistent goals that apply to all students, while encouraging the use of a variety of methods to achieve these goals. This approach should channel the professional energy in each school more fully into instruction - which should result in marked improvements in student achievement in the years ahead.

PHILOSOPHY AND GOALS

The goal of mathematics instruction implicit in this document is the maximum development of students' mathematical and problem-solving power, and the highest possible attainment in the high school mathematics continuum for each student. Accordingly, high school mathematics classes should help students in:

1. Understanding the workings of the decimal number system
2. Understanding some of the patterns into which numbers fall
3. Understanding mathematical terms
4. Maintaining computational skills: addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals
5. Learning how to work accurately and check for errors
6. Estimating answers and testing the reasonableness of results as a way of avoiding errors
7. Using calculators to check work and solve problems when basic concepts and skills have been mastered
8. Maintaining an understanding of units of measurement for time, money, and both English and metric length, capacity, and weight; applying knowledge in practical situations
9. Understanding geometrical shapes, terms, and proofs, and seeing the applications of geometry to the world
10. Following directions and using formulae
11. Interpreting different kinds of graphs, tables, and schedules
12. Understanding and applying basic probability
13. Attacking mathematics problems in a systematic, step-by-step fashion
14. Being able to apply mathematical skills and knowledge to other school subjects and in a variety of other settings
15. Seeing the links between mathematics courses and vocational and avocational applications of mathematics
16. Developing and maintaining a positive attitude toward mathematics based on success, good associations, and relevance

BASIC MATHEMATICS OBJECTIVES

Introduction

The Basic Mathematics Objectives outline a course for students who enter high school without having passed the Massachusetts Basic Skills test in mathematics. The objectives are virtually identical to the Level Six Objectives in the Elementary and Middle School Curriculum document. The rationale for using the Level Six Objectives for Basic Mathematics is to provide high school students who need remedial work in mathematics with the greatest possible degree of continuity with their elementary and middle school experience.

As was pointed out in the general introduction (see pages v and vi), the methods, materials, and emphasis used in teaching these objectives to high school students will be different from those used in middle schools. But the basic goals and objectives are the same - a solid background in computation and other key areas. Mastering these objectives should prepare students to pass the Basic Skills test, and pave the way for success in General Mathematics.

The objectives in Basic Mathematics/Level Six are divided into these eight areas:
A. Numbers and Numeration
B. Computation
C. Fractions
D. Decimals and Percent
E. Measurement
F. Geometry
G. Estimating, Graphs, Etc.
H. Basic Mathematics and Careers

The only changes that have been made from the Level Six Objectives are: (a) minor technical corrections; (b) the circling of some additional objectives as priority items (in response to the feedback received from high school mathematics teachers in June of 1983); and the addition of the Basic Mathematics and Careers section.

For a fuller discussion of the place of these objectives in the overall K-12 mathematics curriculum, see the general introduction of this document, and the introduction and philosophy and goals statement of the Elementary and Middle School Curriculum Objectives.

BASIC MATHEMATICS/
LEVEL SIX

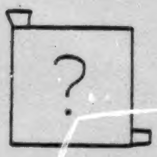
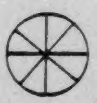
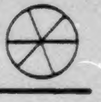
A. NUMBERS AND NUMERATION	
① Writing numerals up to the billions place.	1. Write five billion, six hundred forty-two million, seven hundred thousand in numerals: ____________
2. Using Roman numerals up to two thousand.	2. Write 1982 in Roman numerals: ________ Write MDCCCXLVII in Arabic numerals: ____
③ Identifying place value up to the billions place.	3. In the number 5,672,031,997: How many ten thousands? ________ How many billions? ________ How many tens? ________ How many hundred millions? ________
4. Writing numbers up to billions in expanded notation.	4. Write 4,658,332,091 in expanded notation: ____________________
⑤ Arranging integers in order of value.	5. Arrange these integers in order from the smallest to the largest value: 14 -3 -7 2 0 -18 __ __ __ __ __ __
⑥ Identifying opposite integers.	6. Circle the two opposite integers: 16 -3 46 8 -5 3 -21
7. Continuing a number sequence that rises in arithmetic or geometric steps.	7. Write the next number in each sequence: 0, 4, 8, 12, ____ 2, 4, 8, 16, ____
⑧ Identifying numbers divisible by 2, 3, 4, 5, 9, and 10, and identifying odd and even numbers.	8. Which numbers are divisible by 9? 46 72 135 Which are odd numbers? 4 56 301 705 328
⑨ Identifying prime numbers between one and one hundred.	9. Circle the prime numbers: 28 51 83 91 97 99

BASIC MATHEMATICS/
LEVEL SIX A,B

10. Writing numbers up to one hundred in prime factored form.	Write 28 as a product of prime factors. ________________________
⑪ Writing exponents in words, and vice-versa.	11. Write in words: 4^2 - ________ 8^3 - ________ Write in numeral form: six squared ________ four to the fifth ____
B. COMPUTATION	
① Adding numbers with up to six digits, written vertically or horizontally (if horizontally, students need to stack up numbers correctly).	1. 176,493 + 998,022 + 1,324 = ________
② Subtracting numbers with up to seven digits, written vertically or horizontally (if horizontally, students need to stack up numbers correctly).	2. 8,445,000 - 4,590 = ________
③ Multiplying four-digit numbers by three-digit numbers, written vertically or horizontally.	3. 4,085 x 231 = ________
④ Dividing four-digit numbers by two-digit numbers, with or without remainders, written vertically or horizontally.	4. 2807 - 43 = ______
⑤ Checking for errors in computation at this level.	5. Circle any error or errors: $\begin{array}{r} 781 \\ +390 \\ \hline 1171 \end{array}$ $\begin{array}{r} 875{,}400{,}980 \\ -\ 2{,}334{,}199 \\ \hline 873{,}066{,}781 \end{array}$ $\begin{array}{r} 5401 \\ \times\ 47 \\ \hline 37807 \\ 21604\ \ \\ \hline 253847 \end{array}$ $\begin{array}{r} 66r1 \\ 67\overline{)4432} \end{array}$

BASIC MATHEMATICS/
LEVEL SIX B

⑥ Identifying twelve mathematical terms, given a list of the terms (maintained).	6. Which number is an example of each term? product ____ addend ____ difference ____ quotient ____ sum ____ subtrahend ____ minuend ____ divisor ____ remainder ____ multiplier ____ dividend ____ multiplicand ____ $\begin{array}{r}45\\ \text{x}4\\ \hline 180\end{array}$ $\begin{array}{r}6\\ +6\\ \hline 12\end{array}$ $\begin{array}{r}76\\ -16\\ \hline 60\end{array}$ $\begin{array}{r}\text{2r1}\\ 58\overline{)117}\end{array}$
⑦ Finding the average of a set of two-digit numbers, no remainders, presented in a word problem.	7. A salesman drives 55 miles one week, 31 miles the next, and 97 miles the next. What is the average distance he drives a week? __________
8. Identifying how many times greater one multiple of ten, one hundred, or one thousand is than another, judging by the addition of one, two, or three zeroes.	8. How many times 20 is 20,000? __________
⑨ Working out multiplication and division in parentheses before adding or subtracting (maintained).	9. 35 - (45 - 5) + (4 x 16) = _____
⑩ Evaluating expressions involving two exponents, each of which is 2, [illegible] or 4.	10. $4^2 + 3^3 =$ __________
⑪ Adding and subtracting one-digit integers.	11. $^{+}5 + {}^{-}6 =$ __________
12. Writing open number sentences for word sentences involving one operation.	12. Write a number sentence for this question: Three more than what number is nineteen? ________
⑬ Using letters as place holders in simple open number sentences.	13. 36 - x = 12 x = _____
⑭ Evaluating a simple expression with one variable, given the value of the variable.	14. 12 + b = _____ (b=4)

15. Deducing the one-step rule of a function machine, given input and output information.	15. What is the rule for this machine? ____________________ INPUT | OUTPUT 5 | 12 7 | 14 10 | 17 14 | 21 
16. Finding the missing information needed to solve a word problem that has insufficient information.	16. A plane is flying at 576 miles per hour. How long will it take to reach its destination? What else do you need to know to solve this problem? ________________
17. Solving one- and two-step word problems involving computation at this level, some including extraneous information.	17. David collected 35 baseball cards. Elena collected 70. Each card is worth $5.00. How much are Elena's cards worth? ______
C. FRACTIONS 1. Filling in one-half, one-third, or one-fourth of a figure divided into units that are a sub-multiple of the fraction.	1. Fill in one-half of this circle:  Fill in one-third of this circle: 
2. Circling one-half, one-third, two-thirds, one-fourth, three-fourths, or one-eighth of a row of objects.	2. Circle three-fourths of this group: o o o o o o o o o o o o o o o o
3. Arranging fractions with denominators of twelve and under in order of size.	3. Arrange these fractions in order from smallest to largest: $\frac{1}{5}$ $\frac{1}{2}$ $\frac{3}{8}$ $\frac{4}{9}$ $\frac{1}{3}$ _ _ _ _ _
4. Finding the fraction which when added to a given fraction makes one whole (maintained).	4. 7/11 + / = 1 5/16 + / = 1

BASIC MATHEMATICS/
LEVEL SIX C

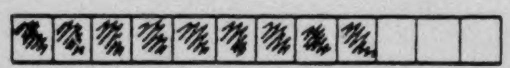
⑤ Adding and subtracting common fractions with unlike denominators, with answers in simplest terms, written horizontally (no negative answers).	5. $\frac{4}{5} + \frac{3}{8} = \_\_$ $\quad \frac{2}{3} - \frac{1}{9} = \_\_$
⑥ Finding a fractional part of a large number, expressed in lowest terms, in word problem form.	6. 350 out of 420 people voted for a candidate in an election. What fraction is this, in simplest terms?
⑦ Adding mixed numbers with like denominators with regrouping, written vertically, with answers in simplest terms.	7. 4 2/3 +2 2/3
⑧ Subtracting mixed numbers with like denominators with regrouping, written vertically, with answers in simplest terms (no negative answers).	8. 6 1/7 -3 3/7
⑨ Adding and subtracting mixed numbers with unlike denominators, no regrouping, written vertically, with answers in simplest terms.	9. 4 1/3 6 1/6 +3 2/5 -4 1/9
⑩ Multiplying two or three common fractions, mixed numbers, or whole numbers, including finding a fraction of money, with simplified answers.	10. 1/3 x 2 3/4 x 4 = ____ 2/3 of $51 = ____
⑪ Dividing common fractions, mixed numbers, or whole numbers, with answers in simplest terms.	11. $\frac{1\ 1/5}{1/2} = \_\_$ $\quad \frac{4/5}{8} = \_\_$
⑫ Identifying numerator, denominator, reciprocal, mixed numbers, and improper fraction without benefit of a list.	12. What is the bottom number in a fraction called? ________ What do you call numbers like $4\frac{1}{2}$ or $2\frac{3}{7}$ What is the reciprocal of 5/6? _____

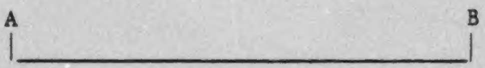
13. Solving ratio and proportion problems in practical situations.	13. One can of concentrated orange juice needs three cans of water. Four cans of concentrate need ____ cans of water.
14. Solving ratio and proportion problems in fraction and ratio form.	14. 3:12 = 6: ____ $\frac{2}{5} = \frac{6}{n}$ n= ____
15. Solving one- and two-step word problems involving fractions at this level.	15. There were 30 problems on a test. Rachel got 5/6 of them right. How many problems did she get wrong? ________

D. DECIMALS AND PERCENT

1. Writing mixed decimals to the ten-thousandths place written as words and numerals.	1. Write five and twenty-eight ten thousandths in numerals: ______________
2. Identifying decimal place value to the ten-thousandths place.	2. The number 3.4902 has: __ hundredths __ tenths __ ten thousandths __ thousandths
3. Arranging decimals in order of size.	3. Arrange these decimals in order from the smallest to the largest. .5 .55 .05 .005 .0555 _____ _____ _____ _____ _____
4. Identifying equivalent decimals when some have redundant zeroes to the right of the last digit (maintained).	4. Which has the same value as .34? _____ .034 .0034 .3400 .00034
5. Writing a number containing millions and hundred thousands in abbreviated decimal notation, given an example from a newspaper story.	5. A newspaper story says that a town's budget is $3.4 million. Write this out in standard form. ______________________
6. Adding and subtracting decimals with different numbers of decimal places, written horizontally (students need to stack up numbers correctly).	6. 55.1 + 3.119 + 600 = ________ $22 - $1.99 = ________

7. Multiplying decimals and whole numbers with up to two-digit multipliers written horizontally or vertically.	7. 4.8 x .35 = ______ 2.15 x 69
8. Dividing decimals and whole numbers with one-digit divisors, written horizontally, without remainders.	8. 1.56 / .3 = ________
9. Writing fractions and mixed numbers with denominators of ten, one hundred, one thousand, and ten thousand as decimals and mixed decimals.	9. Write as decimals and mixed decimals: 1/10,000 = _____; 2 7/100 = _____
10. Changing common fractions to (terminating) decimals.	10. Write the equivalent decimal: 1/4 = _______; 3/8 = _______
11. Identifying simple fractions that have repeating or recurring decimals when converted to decimals.	11. Which of these has a recurring decimal when converted to a decimal? $\frac{1}{2}$ $\frac{3}{8}$ $\frac{2}{3}$ $\frac{7}{8}$ $\frac{1}{4}$
12. Changing decimals to the hundredths place to percents.	12. Change to percents: .34 = _____% .03 = _____%
13. Changing a whole-number percent of two digits to its decimal equivalent.	13. Change to decimals: 82% = _____ 5% - _____

⑭ Changing common fractions to percents (only terminating decimals).	14. Change to percents: 1/2 = _____% 5/8 = _____%
⑮ Changing the percents for reasonably simple common fractions to fractions.	15. Write the fraction for each percent: 30% = _____ 25% = _____ 12 1/2% = _____ 66 2/3% = _____
⑯ Telling what percent of a figure is shaded, giving answer as a reasonably simple common fraction.	16. What percent of this figure is shaded in? _________ 
⑰ Finding the percent of a whole number (only whole number percents).	17. What is 35% of 350? _______
⑱ Solving one- and two-step word problems involving decimals and percents at this level.	18. A meal in a restaurant costs $18.24. What would be a 15% tip on this bill? (Round to the nearest cent.) _____
E. MEASUREMENT	
① Calculating the elapsed time given two points in time in digital or conventional readings (maintained).	1. School begins at 8:45 and lets out at ten minutes past three. How many hours and minutes is school in session?
② Renaming "improper" measures of time (e.g., 25 hours) with two measures.	2. 27 hours = ___ day(s) ____ hour(s) 125 minutes = ___ hour(s) ____ minute(s)

③ Telling the number of days in each month (maintained).	3. How many days are there in March? ____ How many days in September? ____ How many in a leap year February? ____
④ Interpreting and writing dates in numerical form.	4. What month, day, and year does 5/13/82 mean? ____________________ Write November 8th, 1982, in numerical form: ____/____/____
⑤ Using an English and metric ruler to measure a line segment.	5. How long is the line AB in metric measurement? __________ How long is it in English measurement? __________ 
⑥ Renaming "improper" English and metric linear measures with two measures.	6. 25 inches = ____ feet ____ inch(es) 10,000 feet = ____ mile(s) ____ feet 350 centimeters = ____ meter(s) ____ centimeters
⑦ Renaming "improper" English and metric capacity measures with two measures.	7. 9 quarts = ____ gallon(s) ____ quart(s) 3 cups = ____ pint(s) ____ cup(s) 2007 milliliters = ____ liter(s) ____ milliliters
⑧ Renaming "improper" English and metric weight measures with two measures.	8. 35 ounces = ___ pound(s) ___ ounce(s) 3315 grams = ___ kilogram(s) ___ gram(s)
⑨ Applying knowledge of the boiling and freezing points of water and of normal body temperature in degrees Fahrenheit to do simple word problems (maintained).	9. After he ran the Boston Marathon, a man was recorded as having a temperature of 92.3°. How far below normal was this? __________

⑩ Knowing an approximate Celsius temperature reading for a hot day, a mild day, and a cold day, and the exact reading for the boiling and freezing point of water. The following verse is helpful:

Thirty is steamy,
Twenty is nice,
Ten is chilly,
And zero is ice.

10. Write the approximate Celsius temperature (a multiple of ten) for each of these:

* boiling water ______
* a very hot day ______
* a pleasant day ______
* a cold day ______
* freezing water ______

---

⑪ Identifying the appropriate measuring instrument for a task, given a list (maintained).

11. What measuring instrument from the list below would be best for each job?

* measuring a board________
* weighing a letter_________
* counting minutes till lunch_________

(clock, stopwatch, ruler, tape measure, measuring cup, scale, thermometer, calendar)

---

⑫ Identifying the appropriate unit of measurement, English or metric, from a list seprated by English and metric (maintained).

12. Which unit of measurement would be best for each of the following?

	ENGLISH	METRIC
a boulder's weight	________	________
sugar for a cake	________	________
the thickness of a half dollar	________	________

ounces
pounds
tons
cups
pints
quarts
gallons
inches
feet
yards
miles
degrees F.

grams
kilograms
tonnes
milliliters
liters
millimeters
centimeters
meters
kilometers
degrees C.

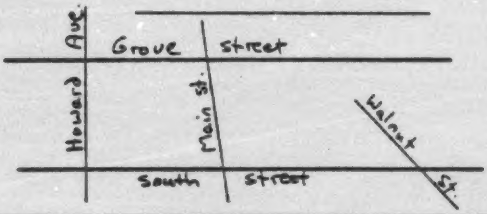
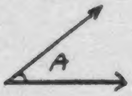
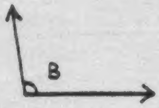
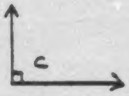
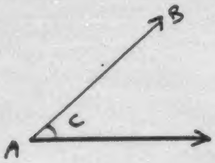
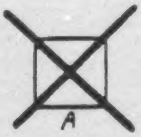
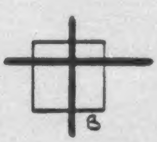
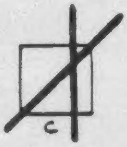
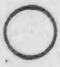
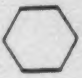
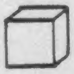
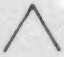
---

⑬ Estimating length, weight, capacity, and time.

13. About how much do you think a full-grown elephant weighs?

100 pounds　　300 pounds　　2 tons

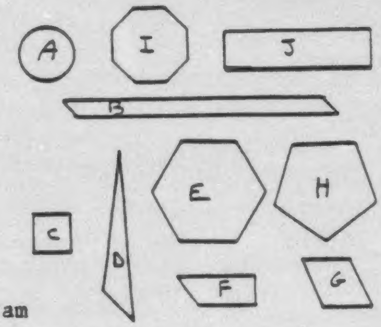
F. GEOMETRY

① Identifying terms dealing with lines, given a map or isolated drawings (a list provided): line segment, point, vertical, horizontal, perpendicular, parallel, and intersecting (maintained).	1. Which street intersects with Walnut Street? ________________ Which street runs perpendicular to South Street? ________________ Which street is parallel to South Street? ________________  Howard Ave. Grove street Main st. Walnut St. South street
② Identifying right, acute, and obtuse angles, given drawings and a list (maintained).	2. Which is a right angle? ____; an acute angle? ____; an obtuse angle? ____   
③ Identifying the terms ray, vertex, and angle in a drawing (maintained).	3. Write the letter for each term: ___ a ray ___ the vertex ___ the angle 
4. Identifying a figure with two lines of symmetry drawn through it.	4. Which square has two lines of symmetry drawn through it? ____   
5. Classifying shapes by whether they are one-, two-, or three-dimensional.	5. Write by each figure whether it is one-, two-, or three-dimensional:  ____  ____  ____  ____

⑥ Identifying ten two-dimensional figures (circle, square, triangle, rectangle, parallelogram, rhombus, trapezoid, pentagon, hexagon, octagon, given a list [maintained]).

6. Write the letter for each shape by its name:

___ pentagon
___ trapezoid
___ triangle
___ square
___ octagon
___ circle
___ rectangle
___ rhombus
___ hexagon
___ parallelogram



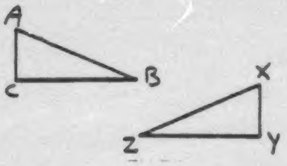
⑦ Identifying where these shapes appear in the real world (maintained).

7. What shape is a stop sign?

* octagon
* hexagon
* parallelogram

8. Identifying congruent sides and angles in congruent figures.

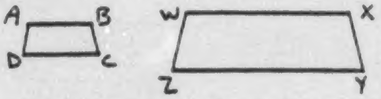
8. If these two triangles are congruent, which angles and sides are equal?



Δ ABC	XYX
< ABC	
< ACB	
$\overline{AB}$	
$\overline{AC}$	

9. Identifying corresponding sides and congruent angles in similar triangles.

9. If these two trapezoids are similar, which angles are equal?



ABCD	WXYZ
< ABC	
< ADC	
< BCD	

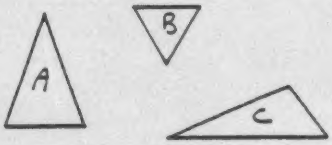
⑩ Identifying isosceles, equilateral, and scalene triangles.

10. Write the letter of each kind of triangle by its name:

___ equilateral

___ isoceles

___ scalene



⑪ Finding the perimeter of a square, rectangle, or triangle in a word problem.

11. A man wants to put a fence around a rectangular garden 15 feet long and 12 feet wide. How many feet of fence does he need? __________

BASIC MATHEMATICS/
LEVEL SIX F

⑫ Finding the area of a rectangle or square in a word problem, given the formula, with answers in square units.

12. A woman wants to buy carpet to cover the floor of a room 12 feet long and 8 feet wide. How many square feet of carpet will she need?

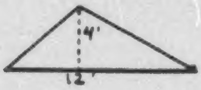
____

(Area = length x width)

⑬ Finding the area of any triangle or parallelogram, given the formula, with answers in square units.

13. What is the area of this triangle?

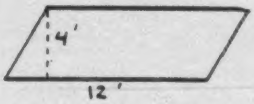
____



(Area = 1/2 base x height)

What is the area of this parallelogram?

____

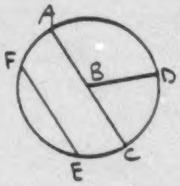


(Area = base x height)

⑭ Identifying center, diameter, chord, radius, and circumference of a circle.

Write the letters for each part of this circle:

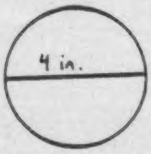
____ diameter
____ radius
____ center
____ chord
____ circumference



⑮ Finding the circumference of a circle, given the formula and the value of #.

15. What is the circumference of this circle?

____

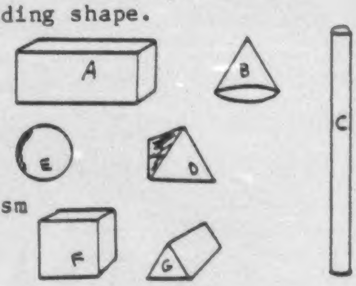


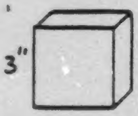
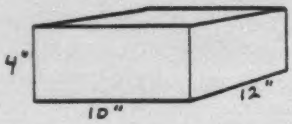
(Circumference = π x diameter)
Use π = 3.14

⑯ Identifying seven geometric solids (cube, rectangular solid, cylindler, cone, sphere, pyramid, and triangular prism), and everyday objects that have these shapes, given a list.

16. Write the letter for each shape by its name or corresponding shape.

__ pyramid
__ sphere
__ cube
__ cone
__ cylinder
__ brick
__ triangular prism

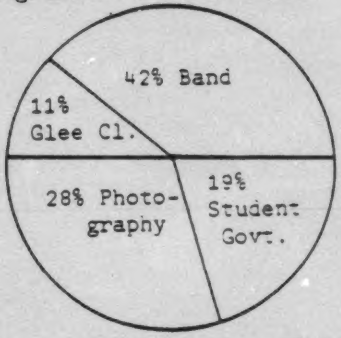


17. Finding the surface area of a cube or rectangular solid, given the formula.	17. What is the surface area of this cube? (surface area = 6 x E^2, where E is the length of each edge)  __________
⑱ Finding the volume of a rectangular solid or cube, given the formula, with answers in cubic units (maintained).	18. What is the volume of this box? ________ (volume = length x width x height) 
G. ESTIMATING, GRAPHS, ETC. ① Rounding whole numbers to any place up to hundred thousands.	1. Round off to the nearest thousand: 34,620 ________ Round off to the nearest hundred thousand: 255,029 ________
② Rounding decimals and mixed decimals to the nearest whole number, tenth, or hundredth, and money to the nearest cent.	2. Round off to the nearest tenth: .862231 ________ Round off to the nearest cent: $14.29332 ________
③ Estimating answers to word problems by rounding, including area and volume problems.	3. A man made $19,638.51 one year. About how much would he make in three years at this salary? $40,000 $50,000 $60,000 $80,000
④ Identifying probability in word problems.	4. Julie has a bag with 3 blue marbles and 2 green marbles. If she reaches into the bag without looking, what are her chances of pulling out a blue marble? 2 out of 3 2 out of 5 3 out of 5

BASIC MATHEMATICS/
LEVEL SIX G

⑤ Using picture, bar, line, and circle graphs to answer questions and solve problems.

5. This graph shows the percent of students taking different electives.



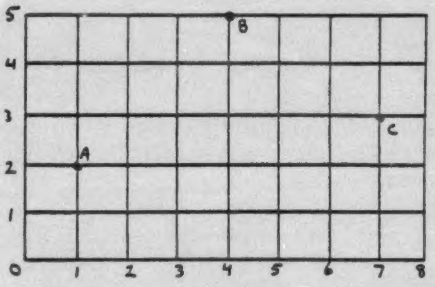
What percent chose Glee Club?

How much greater was the percent taking Band than Photography?

⑥ Graphing ordered pairs, and giving the coordinates of plotted points.

Write coordinates for each point on the grid:

A ____ B ____ C ____



⑦ Identifying the median of a set of data.

7. What is the median of this set of test scores?

100% 85% 85% 93% 42% 66% 48%

⑧ Getting information from a table or schedule.

8. Which city reached its peak in 1970? ________________ Which city lost the most people from its peak? ________________

Population of Several U.S. Cities 1920 - 1970

	1920	1940	1960	1970
Boston	748,060	770,816	697,197	641,071
Detroit	993,678	1,623,144	1,670,144	1513,601
Houston	138,276	384,514	938,219	1232,802
San Francisco	506,676	634,536	740,316	715,674

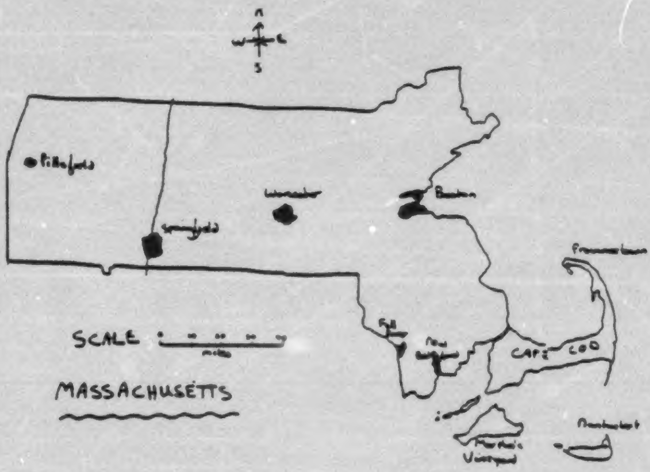
⑨ Getting distance information from a map by using the scale.

9. Use this map and the scale to estimate the distance by air from:

Boston to Provincetown __________

Boston to Pittsfield __________

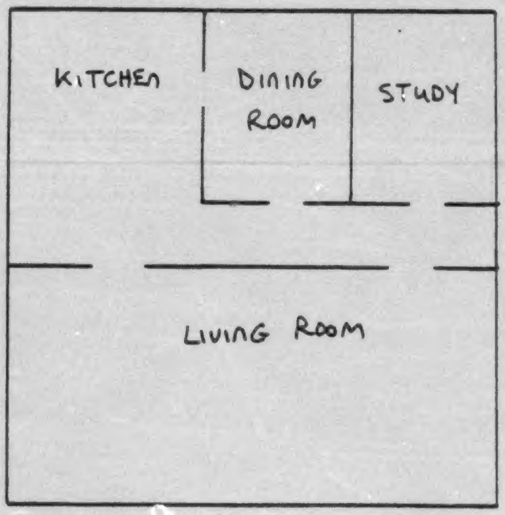
Springfield to Nantucket __________


N
W
E
S
Pittsfield
Worcester
Springfield
Boston
Provincetown
CAPE COD
Nantucket
SCALE
miles
MASSACHUSETTS

⑩ Getting information from a scale drawing.

10. Use your ruler to find the dimensions of the living room in this scale drawing.

__________ long __________ wide


KITCHEN
DINING ROOM
STUDY
LIVING ROOM

SCALE: 1 in. = 10 ft.

H. BASIC MATHEMATICS AND CAREERS

Identifying jobs in which each of the following plays an important role:

-Numbers
-Computation
-Fractions
-Decimals and Percent
-Measurement
-Geometry
-Estimating and Graphs

GENERAL MATHEMATICS OBJECTIVES

Introduction

The General Mathematics Curriculum Objectives outline a course for students who have passed the Massachusetts Basic Skills test in mathematics but are not yet ready to take Algebra I. The objectives are virtually identical to the Level Seven Objectives in the Elementary and Middle School Curriculum document. The rationale for using the Level Seven Objectives for General Mathematics is to provide high school students below the Algebra I level with the greatest possible degree of continuity with their elementary and middle school experience.

As was pointed out in the general introduction (see pages v and vi), the methods, materials, and emphasis used in teaching these objectives to high school students will be different from those used in middle schools. But the basic goals and objectives are the same - a solid background in computation and other key areas. Mastering these objectives should prepare students to take Algebra I should they so choose.

The objectives in General Mathematics/Level Seven are divided into these eight areas:
A. Numbers and Numeration
B. Computation
C. Fractions
D. Decimals and Percent
E. Measurement
F. Geometry
G. Estimating, Graphs, Etc.
H. General Mathematics and Careers

The only changes that have been made from the Level Seven Objectives are: (a) minor technical corrections; (b) the circling of some additional objectives as priority items (in response to the feedback received from high school mathematics teachers in June of 1983); and the addition of the General Mathematics and Careers section.

For a fuller discussion of the place of these objectives in the overall K-12 mathematics curriculum, see the general introduction of this document, and the introduction and philosophy and goals statement of the Elementary and Middle School Curriculum Objectives.

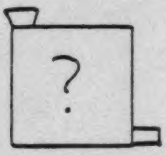
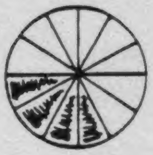
GENERAL MATHEMATICS/
LEVEL SEVEN

A. NUMBERS AND NUMERATION	
① Writing numerals up to the hundred trillions place.	1. Write fifty-nine trillion, four hundred billion, seventy-three thousand in numerals: ____________________
2. Using Roman numerals up to two thousand (maintained).	2. Write 1496 in Roman numerals: ________ Write MCMDCCCXLIV in Arabic numerals: ___
③ Identifying place value up to the trillions place.	3. In the number 6,538,204,191,000: How many trillions? ____ How many ten thousands? ____ How many billions? ____
④ Identifying a ten-digit number from its expanded exponential notation.	4. Write in standard form: $(9 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) +$ $(7 \times 10) + (6 \times 1)$ ______________________________
⑤ Arranging any integers in order. •	5. Arrange these integers in order from the smallest to the largest value: 46 -3 -27 552 0 34 -34 ___ ___ ___ ___ ___ ___ ___
⑥ Identifying opposite integers.	6. Circle the two opposite integers: 45 -24 98 -54 24 51 -100
⑦ Continuing a number sequence that rises in arithmetic or geometric steps (maintained).	7. What are the next two numbers in this sequence? 2, 4, 8, 16, ____, ____.
⑧ Identifying numbers divisible by 2, 3, 4, 5 ,9, and 10, and identifying odd and even numbers (maintained).	8. Which numbers are divisible by 4? 34 51 92 12 Which are even numbers? 4 290 391 1000

GENERAL MATHEMATICS/
LEVEL SEVEN A,B

⑨ Identifying prime numbers up to one hundred (maintained).	9. Write the prime numbers between 70 and 80: ________
⑩ Expressing a number up to one hundred as a product of prime factors, using exponential notation when necessary.	10. Write 80 as a product of prime factors, using exponential notation: ________
⑪ Expressing exponents in words, and vice-versa (maintained).	11. Write in words: 12^5 ________ 6^3 ________ Write in numeral form: nine squared ________ two to the sixth ________
⑫ Writing products of 10 as exponential powers of 10.	12. Write as powers of 10: 10,000 ________ 1,000,000 ________
⑬ Finding the square roots of perfect squares.	13. What is the square root of 64? ________ of 144? ________
14. Estimating the square root of a nonperfect square number under one hundred, to the nearest tenth.	14. What is the square root of 80, rounded to the nearest tenth? ________
B. COMPUTATION ① Adding numbers with up to seven digits, written vertically or horizontally (if horizontally, students need to stack up numbers correctly).	1. 2,335,545 + 329,009 + 44,391 = ________
② Subtracting numbers with up to eight digits, written horizontally or vertically (if horizontally, students need to stack up numbers correctly).	2. 54,000,000 - 3,452,993 = ________
③ Multiplying five-digit by three-digit numbers, written vertically or horizontally.	41,398 x 981 = ________

④ Dividing five-digit by three-digit numbers, with remainders, written vertically or horizontally, with quotients expressed as mixed numbers.	4. 89,284 / 132 = ____________ (express the answer as a mixed number).
⑤ Checking for errors in computation at this level.	5. Circle any error or errors: $\begin{array}{r} 86694 \\ 29650 \\ +3298 \\ \hline 102242 \end{array}$ $\begin{array}{r} 30000 \\ -2572 \\ \hline 32572 \end{array}$ $\begin{array}{r} 8391 \\ \times 315 \\ \hline 41955 \\ 8391 \\ 25173 \\ \hline 2643265 \end{array}$ $\begin{array}{r} 514\ 15/92 \\ 92\overline{)47293} \end{array}$
6. Using the twelve terms for different parts of computation problems without benefit of a list (addend, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, divisor, dividend, quotient, remainder).	6. What is the answer to a division problem called? ____________ What is the top number in a subtraction problem called? ____________
⑦ Finding the average of whole numbers and decimals presented in word problem form, rounding remainders to the nearest unit.	7. What is the average of these test scores: 88%, 47%, 93% and 69%? (Round to the nearest whole percent.) ____________
8. Identifying how many times greater one multiple of ten, one hundred, or one thousand is than another, judging by the addition of one, two, or three zeroes (maintained).	8. 5,600 is how many times as great as 56?
⑨ Knowing the order of operations in expressions written without parentheses (multiplication and division before addition and subtraction).	9. 5 + 3 x 12 - 3 x 6 = _____
⑩ Solving problems with two or three exponents up to the fifth power.	10. $5^2 + 3^4 + 9^3 =$ _____
⑪ Adding and subtracting one-digit integers (maintained).	11. $^-6 + {}^-4 =$ _____

⑫. Writing open number sentences for word sentences involving one operation (maintained).	12. Write the number sentence for this question: Seven times what number is eighty-four? ________
⑬. Solving a simple equation with one variable and one operation.	13. a + 15 = 25; a = ____
⑭. Finding the value of an open phrase with two variables, given the value of the variables.	14. 4a + 3b = ____ (a = 2, b = 6)
15. Deducing the two-step rule of a function machine, given input and output information.	15. What is the rule for this machine? ________ INPUT | OUTPUT 1 | 5 2 | 8 3 | 11 
⑯. Identifying the missing information needed to solve a word problem that has insufficient information.	16. A farmer's chickens produced 5,411 eggs one year and 3,492 eggs the next. How many eggs did each chicken produce for the two years? What additional information do you need to solve this problem? ________
⑰. Solving one-, two-, or three-step word problems involving computation at this level, some including extraneous information.	A factory turned out 350 cars each month. The average value of each car was $6,344. At this rate, what would be the value of the cars produced by the factory in a year?
C. **FRACTIONS**	
①. Writing the common fraction that is shaded reduced to its lowest terms.	1. What fraction of the circle is shaded? State the answer in the lowest terms. ________ 

② Circling a fraction of a row of objects.	2. Circle 5/8 of this group: ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○
③ Arranging fractions in order of size.	3. Arrange these fractions in order from smallest to largest: 1/2 3/4 9/10 7/8 1/3 4/15 ____ ____ ____ ____ ____ ____
4. Finding the fraction halfway between two fractions.	4. What fraction is halfway between 1/8 and 4/5? ________
⑤ Finding the fraction which when added to a given fraction makes one whole (maintained).	5. 7/17 + / = 1
⑥ Finding a fractional part of a large number, expressed in lowest terms, in word problem form (maintained).	6. A man earns $27,000 a year. He spends $18,000. What fraction of this income did he spend, reduced to the simplest terms? ________________
⑦ Adding two or three mixed numbers with unlike denominators with regrouping, written vertically or horizontally, giving answers in simplest terms.	7. $\begin{array}{r} 2\ 5/7 \\ \underline{+4\ 2/3} \end{array}$ 4 1/3 + 2 1/8 + 5 3/8 = _______
⑧ Subtracting mixed numbers with unlike denominators with regrouping, written vertically, giving answers in simplest terms.	8. $\begin{array}{r} 5\ 1/8 \\ \underline{-2\ 4/5} \end{array}$
⑨ Multiplying two or three common fractions, mixed numbers, or whole numbers, including finding a fraction of money, giving simplified answers (maintained).	9. 4 1/5 x 5 2/3 x 7 = _____ 1/4 of $120 = ____

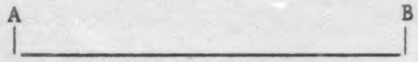
⑩ Dividing common fractions, mixed numbers, or whole numbers, with answers in simplest terms (maintained).	10. $\frac{7\ 2/3}{2\ 1/2}$ = ______
⑪ Identifying numerator, denominator, reciprocal, mixed number, and improper fraction without benefit of a list (maintained).	11. In the fraction 1/4, what is the top number called? ______________ What is the reciprocal of 1/4? ________ Which of these is a mixed number? 43/8 2 1/4
⑫ Solving ratio and proportion problems in practical situations (maintained).	12. An aging motorcycle uses two quarts of oil for three gallons of gasoline. How many quarts of oil would be needed for nine gallons of gas? ____________
⑬ Solving ratio and proportion problems in fraction and ratio form (maintained).	13. 8/9 = 64n, n = ____; 8:48 = 12: ___
⑭ Solving one- and two-step word problems involving fractions at this level.	14. A woman mixed 1/3 of a gallon of paint with 1/2 of a gallon and then added another 1/4 of a gallon. How much paint does she have now? ____________
D. DECIMALS AND PERCENT ① Writing mixed decimals to the millionth place as numerals.	1. Write two and fifteen millionths in numerals: ______________
② Identifying decimal place value to the millionth place.	2. In the number 4.5087193: How many hundredths? ____ How many ten thousandths? ____ How many tenths? ____ How many millionths? ____

GENERAL MATHEMATICS/
LEVEL SEVEN D

③ Arranging decimals in order of size (maintained).	3. Arrange these decimals in order from the smallest to the largest. .024 .24 .204 .0024 _____ _____ _____ _____
④ Identifying equivalent decimals when some have redundant zeroes to the right of the last digit (maintained).	4. Which has the same value as .901? _____ .90100 .091 .00901 .109
⑤ Writing a number containing billions and hundred millions in abbreviated decimal notation, given an example from a newspaper story.	5. A newspaper story says that a military weapons program would cost $1.5 billion. Write this out in standard form: _____________
⑥ Adding and subtracting decimals and mixed decimals with different numbers of decimal places, written horizontally (students need to stack numbers up correctly [maintained]).	6. 7.54 + 998.02 + 56 = _____________ 5.003 = .9801 = _____________
⑦ Multiplying decimals, mixed decimals, and whole numbers, some problems needing one or more zeroes to the left of the product, written vertically or horizontally.	7. .164 x .032 = _____________
⑧ Dividing decimals, mixed decimals, and whole numbers with two-digit divisors, some with additional zeroes required in the dividend, with answers rounded to a specific place.	8. 65.8 / .0032 = _______ (to the nearest tenth)
⑨ Dividing by ten, one hundred, or one thousand by moving the decimal point.	9. Divide 3.95 by 100 by moving the decimal point. _____________
⑩ Writing fractions and mixed fractions with denominators of ten, one hundred, one thousand, and so on to one million, as decimals and mixed decimals.	10. Write as decimals and mixed decimals: 7/1,000,000 = _____________ 9 3/100,000 = _____________

GENERAL MATHEMATICS/
LEVEL SEVEN D

⑪ Changing any common fraction to a decimal (to a given number of decimal places).	11. Write 7/9 as a decimal, rounded to three decimal places: ________________
⑫ Changing decimals to the thousandths place and mixed decimals to percents.	12. Change to percents: 1.3 = ____% .333 = ____%
⑬ Changing percents with one decimal place and percents over one hundred to decimals and mixed decimals.	Change to decimals: 155% = ______; .051% = ______
⑭ Changing common fractions with terminating and recurring decimals to percents.	14. Change to percents, rounded to the tenths place: 1/3 = ____% 5/9 = ____%
⑮ Memorizing the percent equivalencies of frequently used fractions (one-half, one-third, two-thirds, one-quarter, all fifths, eighths, and tenths).	15. Fill in these percent equivalencies: 1/4 = ___%; 4/5 = ___%; 7/10 = ___%
⑯ Finding the percent of a whole number or mixed decimal when the percent is a mixed decimal or more than one hundred.	16. What is 9 1/2% of 2500? ____________ What is 150% of 490? ____________
⑰ Finding what percent one number is of another (percents under one hundred), expressed in word problems.	17. A boy got 15 out of 24 right on a math test. What percent was that? ________
⑱ Finding a number when the percent of it is known (percents under one hundred).	18. 16 is 80% of what number? __________
⑲ Solving one- and two-step word problems involving decimals and percents at this level, including problems with commissions, discounts, profit, interest, etc.	19. A car cost $7,395 after being marked down from $8,700. What was the percent of the discount? ________________ How much would a $55 dress cost if it were marked down 15%? ____________

E. MEASUREMENT	
① Calculating elapsed time, given two points in time in digital or conventional readings (maintained).	1. A concert begins at 8:45. It is now ten minutes before six. How much time is there before the concert? ____________
② Doing two-step conversions between units of time (seconds, minutes, hours, days, weeks, years, decades, centuries).	2. How many seconds are there in two hours? ____________ How many hours in two weeks? ________
③ Telling the number of days in each month (maintained).	3. How many days are there in June? _____ How many days in a non-leap year February? ____________
④ Interpreting and writing dates in numerical form (maintained).	4. What month, day, and year does 8/2/83 mean? __________________________ Write February 14, 1983, in numerical form: _____/_____/_____
⑤ Using an English and metric ruler to measure a line segment (maintained).	5. How long is the line AB in inches? ____ How long in metric measurement? _______ 
⑥ Doing two-step conversions within English linear measures and within metric linear measures (inches, feet, yards, miles; millimeters, centimeters, meters, kilometers).	6. How many inches are there in two miles? ___________ How many millimeters in a kilometer? ___________
⑦ Doing two-step conversions within English capacity measures and within metric capacity measures (cups, pints, quarts, gallons; milliliters, liters, kiloliters).	7. How many cups are there in three quarts? ________________________________ How many milliliters in six kiloliters? ________________________________

<table>
<tr><td>⑧ Doing two-step conversions within English measures of weight and within metric measures of weight (ounces, pounds, tons; grams, kilograms).</td><td>8. How many ounces are there in two tons? ________
How many grams in 15 kilograms? ________</td></tr>
<tr><td>⑨ Applying knowledge of the boiling and freezing points of water and of normal body temperature in degrees Fahrenheit to do simple word problems (maintained).</td><td>9. A woman ran a fever 5.8° above her normal body temperature. What was her temperature during the fever? ________</td></tr>
<tr><td>⑩ Knowing an approximate Celsius temperature reading for a hot day, a mild day, and a cold day, and the exact reading for the boiling and freezing point of water (maintained). The following verse is helpful:
Thirty is steamy,
Twenty is nice,
Ten is chilly,
And zero is ice.</td><td>10. Write the Celsius temperature (a multiple of ten) for each of these:
a chilly day ____
freezing water ____
a very hot day ____
a pleasant day ____
boiling water ____</td></tr>
<tr><td>⑪ Identifying the appropriate measuring instrument for a task, given a list (maintained).</td><td>11. What measuring instrument from the list below would be best for each job?
weighing a letter: ________
finding a young baby's age: ________
measuring milk for pancakes: ________
(clock, stopwatch, ruler, tape measure, measuring cup, postal scale, bathroom scale, thermometer, calendar)</td></tr>
<tr><td>⑫ Identifying the appropriate unit of measurement, English or metric, from a list separated by English and metric (maintained).</td><td>12. Which unit of measurement would be best for each of the following?
<table>
<tr><th></th><th>ENGLISH</th><th>METRIC</th></tr>
<tr><td>a child's weight</td><td></td><td></td></tr>
<tr><td>length: piece of paper</td><td></td><td></td></tr>
<tr><td>soup in a can</td><td></td><td></td></tr>
</table></td></tr>
</table>

ENGLISH	METRIC
ounces	grams
pounds	kilograms
tons	tonnes
cups	milliliters
quarts	kiloliters
gallons	millimeters
inches	centimeters
feet	meters
yards	kilometers
miles	degrees C
degrees F	

⑬ Estimating length, weight, capacity, and time.

13. About how long do you think it takes to walk a mile?

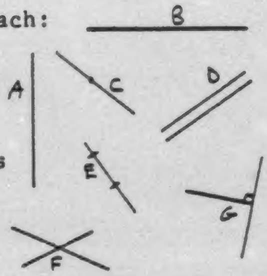
20 minutes   2 hours   5 hours

F. GEOMETRY

① Identifying terms dealing with lines, given a map or isolated drawings (a list provided): line segment, point, vertical, horizontal, perpendicular, parallel, and intersecting (maintained).

1. Write the letter for each:

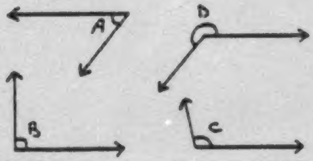
___ point
___ line segment
___ parallel lines
___ perpendicular lines
___ intersecting lines
___ horizontal line
___ vertical line



② Identifying right, acute, obtuse, and reflex angles, given a list.

2. Write the letter for each:

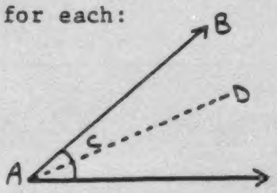
___ acute angle
___ right angle
___ reflex angle
___ obtuse angle



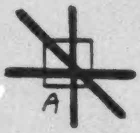
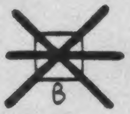
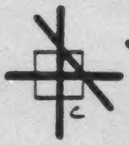
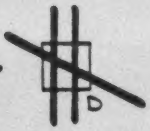
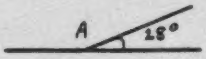
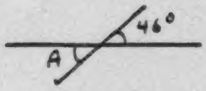
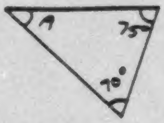
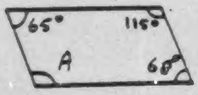
③ Identifying a ray, vertex, angle, and bisector of an angle.

3. Write the letters for each:

___ angle
___ ray
___ vertex
___ bisector of an angle



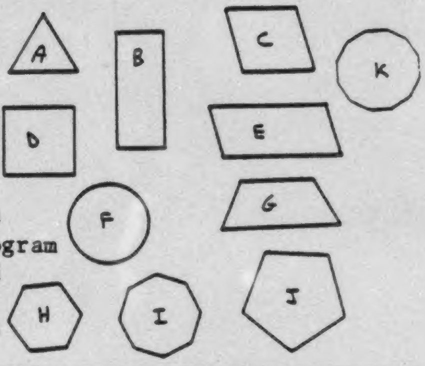
GENERAL MATHEMATICS/
LEVEL SEVEN F

4. Identifying a figure with two or more lines of symmetry drawn through it.	4. Which figure has three lines of symmetry drawn through it?    
5. Finding vertical and supplementary angles.	5. What is the missing angle in each figure?   A = _____; A = _____
⑥ Finding a missing angle in a triangle and a quadrilateral, given the other angles.	6. What is the missing angle in each figure?   A = _____; A = _____
⑦ Distinguishing the differences and overlaps among the terms quadrilateral, square, rectangle, parallelogram, and trapezoid.	7. Write true or false after each statement: All quardrilaterals are squares. _____ All squares are quadrilaterals. _____ All rectangles are parallelograms. _____ All parallelograms are rectangles. _____ Some parallelograms are squares. _____

⑧ Identifying polygons with up to twelve sides.

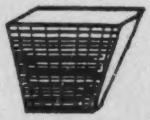
Write the letter for each shape next to its name.

__ hexagon
__ triangle
__ circle
__ square
__ pentagon
__ rhombus
__ rectangle
__ parallelogram
__ trapezoid
__ octagon
__ dodecagon



⑨ Identifying where these shapes appear in the real world (maintained).

9. What shape is the roof of the John Hancock building?

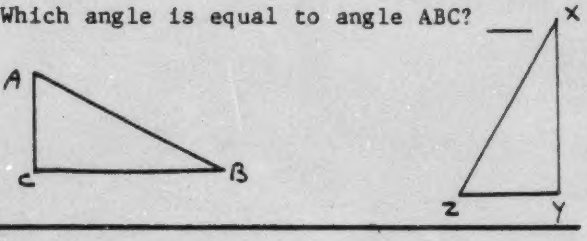


10. Identifying the congruent sides and angles in congruent figures (maintained).

10. Triangle ABC ≅ triangle XYZ.

Which side is equal to AB? ___

Which angle is equal to angle ABC? ___

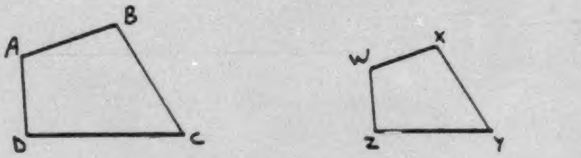


11. Identifying corresponding sides and congruent angles in similar polygons (maintained).

11. ABCD is similar to WZYZ.

Which angle is equal to ABC? ___

Which side in WXYZ corresponds to BC? ___



⑫ Identifying the characteristics of an isosceles, scalene, and equilateral triangle.

12. Write the name isosceles, scalene, or equilateral by each description:

a triangle with all sides equal ________
[Note: two possible answers, isosceles and equilateral.]
a triangle with two sides equal ________

a triangle with three unequal sides ____

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⑬ Finding the perimeter of any figure, given a word problem.

13. A man wants to nail molding around the edge of the ceiling of a rectangular room 14 feet long and 11 feet wide. How many feet of molding will he need? __________

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⑭ Finding the area of a rectangle or a square in a word problem, given the formula, with answers in square units (maintained).

14. A woman wants to spread fertilizer on a lawn 25 feet long and 23 feet wide. How many square feet does she have to cover?

____________ (A=lw)

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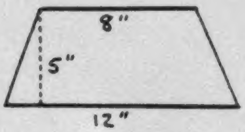
⑮ Finding the area of any triangle, parallelogram, or trapezoid, given the formula, with answers in square units.

15. What is the area of this trapezoid?



____

$A = (b + a)/2 \times h$

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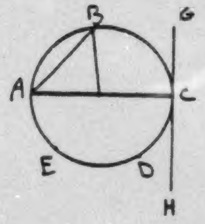
⑯ Identifying center, diameter, radius, chord, arc, circumference, and tangent of a circle.

16. Write the name of each part of the circle by the letters:

diameter
radius
circumference
tangent
chord
arc
center



BO ______ O ______ ED ______ AB ______

ED ________ ABCDE ________ GCH ________

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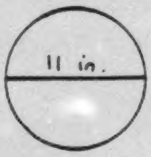
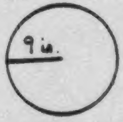
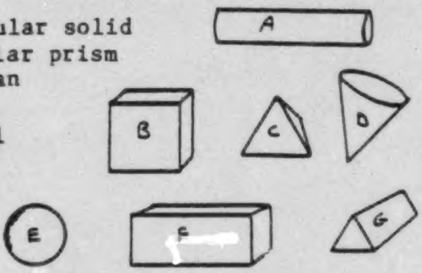
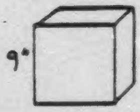
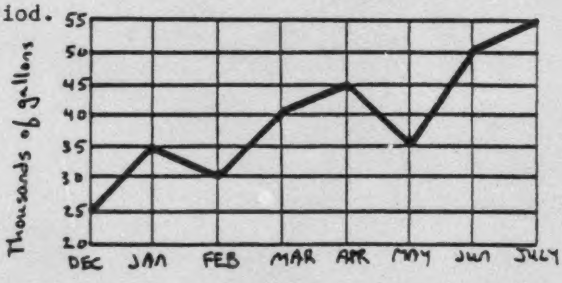
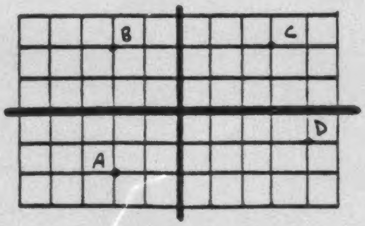
⑰ Finding the circumference of a circle, given the formula and the value of pi (maintained).

17. What is the circumference of this circle?



____
($C = \pi d$;
$\pi = 3.14$)

18. Finding the area of a circle, given the formula and the value of , with answers in square units.	18. What is the area of a circle whose radius is 9 inches?  __________ ($A = \pi r^2$; $\pi = 3.14$)
19. Identifying seven geometric solids (cube, rectangular solid, cylinder, cone, sphere, pyramid, and triangular prism), and everyday objects that have these shapes (maintained).	19. Write the letter for each shape by its name or corresponding object. __ rectangular solid __ triangular prism __ tonic can __ pyramid __ baseball __ cube __ cone 
20. Finding the surface area of solid figures, given the formula, with answers in square units.	20. What is the surface area of this cube?  __________ ($SA = 6 \times E^2$)
21. Finding the volume of a rectangular solid or cube in a word problem, given the formula, with answers in cubic units.	21. What is the volume of a shoe box 12 inches long, 4 inches wide, and 3 inches deep? __________ [$V = (l) \times (w) \times (h)$]
G. ESTIMATING, GRAPHS, ETC.	
1. Rounding off whole numbers to any place up to billions.	1. Round off to the nearest billion: 34,566,199,027 __________
2. Rounding off decimals or mixed decimals to any place up through the thousandths.	2. Round off to the nearest thousandth: 45.983666 __________

③ Estimating answers to word problems by rounding off, including area and volume problems (maintained).	3. A woman's resturant bill came to $21.04. If she wanted to leave a tip of 15%, about how much would the tip be? ______
④ Identifying probability in word problems.	4. A man buys three lottery tickets. He finds out that 15,000 tickets in all were sold. What are his chances of winning? 1 in 10; 1 in 100; 1 in 5,000; 1 in 15,000.
5. Working with set terminology: union and intersection, null and empty, finite and infinite, etc., in numerical form or Venn diagrams.	5. A {5, 10, 15, 20 . . .} B {1, 2, 3, 4, 5} What is the union of sets A and B? ______ What is the intersection? ______ Which set is infinite? ______
⑥ Using picture, bar, line, and circle graphs with multiples on one coordinate to answer questions and solve problems.	6. This is a graph of the gasoline consumed by a fleet of trucks in an eight-month period.  How much gasoline was used in June? ______ In which months were 35,000 gallons used?
⑦ Plotting points on a Cartesian plane.	7. Which point has the coordinates (4,-1)? ______ 

H. GENERAL MATHEMATICS AND CAREERS

Examining one job from each of two occupational clusters in which each of the following plays an important role:

- -Numbers and Numeration
- -Computation
- -Fractions
- -Decimals and Percent
- -Measurement
- -Geometry
- -Estimating and Graphing

PRE-ALGEBRA OBJECTIVES

Introduction

The objectives in this section are taken directly from Levels Six, Seven, and Eight of the Elementary and Middle School Mathematics Curriculum Objectives. They do not spell out a full course, but rather a set of minimum prerequisites which students need to have mastered to experience success in Algebra I. These objectives can be used as a checklist or as review for students who have completed General Mathematics, Level Seven, or Level Eight.

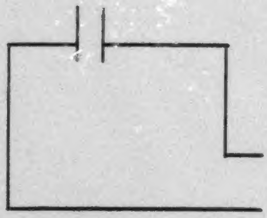
The Pre-Algebra Objectives are divided into the following sections:
A. Numbers and Numeration
B. Computation
C. Fractions
D. Decimals and Percent
E. Estimating, Graphs, Etc.

It should be stressed that these objectives are not a balanced general mathematics course, lacking as they do a geometry and measurement section and much of the substance of the upper levels of the Elementary and Middle School Curriculum Objectives. Students who need a full year of high school mathematics prior to Algebra I should take courses based on the General Mathematics Objectives which precede this section.

PRE-ALGEBRA

A. NUMBERS AND NUMERATION	
1. Continuing number sequences in which each step doubles the previous one.	1. What are the next two numbers in this sequence? 2, 4, 8, 16, ___, ___
2. Identifying opposite integers.	2. Circle the two opposite integers: 16 -3 46 8 -5 3 -21
3. Continuing a number sequence that rises in arithmetic or geometric steps.	3. Write the next number in each sequence: 0, 4, 8, 12, ___ 2, 4, 16, ___
4. Identifying prime numbers between one and 100.	4. Circle the prime numbers: 28 51 83 91 97 99
5. Arranging any integers in order.	5. Arrange these integers in order from the smallest to the largest value: 46 -3 -27 552 0 34 -34 ___ ___ ___ ___ ___ ___ ___
6. Writing products of ten as exponential powers of ten.	6. Write as powers of 10: 10,000 = ___ 1,000,000 = ___
7. Finding the square root of perfect squares.	7. What is the square root of 64? ___ ...of 144? ___
8. Estimating the square root of a nonperfect square number under 100, to the nearest tenth.	8. What is the square root of 80, rounded to the nearest tenth? ___
9. Writing numbers in scientific notation, and vice-versa.	9. Write in scientific notation: 45,652,000,000 ________________ Write in standard notation: $(6.1 \times 10^{9}) + (8.45 \times 10^{6})$ __________

PRE-ALGEBRA A,B

10. Giving the absolute value of a number.	10. Which two integers have the same absolute value? -3 2 45 3 65 -32
11. Expressing a number up to 200 as a product of prime factors, using exponential notation.	11. Write the prime factors of 144, using exponential notation: ____________
B. COMPUTATION	
1. Writing open number sentences for word sentences involving one operation.	1. Write a number sentence for this question: Three more than what number is nineteen? ________________
2. Using letters as place holders in simple open number sentences.	2. 36 ÷ x = 12; x = ___
3. Deducing the one-step rule of a function machine, given input and output information.	3. Look at the input and output of this function machine and figure out what its rule is. _______ INPUT | OUTPUT 5 | 12 7 | 14 10 | 17 14 | 21 
4. Identifying integers and whole, rational, irrational, and real numbers.	4. Which of the numbers below is an: integer ____________ whole number __________ rational number __________ irrational number __________ real number ___________ 3.5 6 -1 0 $\frac{2}{3}$ π $\sqrt{3}$ 9.3 $4.3\overline{3}$

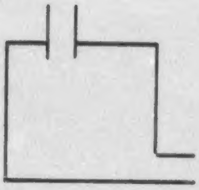
PRE-ALGEBRA B

5. Applying the order of operations in expressions written without parentheses, using the acronym Please Excuse My Dear Aunt Sally (parentheses, exponents, multiplication, division, addition, and subtraction).	5. $3 + 4 \times 8 - 3^2 \times 4 =$ ____
6. Solving exponent problems with three or more parts and higher powers.	6. $8^3 + 7^2 + 2^7 =$ ____
7. Finding the product in exponential form of two or more positive integral powers with the same base.	7. Give the answer in exponential form: $7^2 \cdot 7^4 \cdot 7^3 =$ ____
8. Finding the product in exponential form of two or more positive and negative integral powers with the same base.	8. Give the answer in exponential form: $5^{-6} \cdot 5^2 =$ ____
9. Finding the quotient in exponential form of a positive and negative integral power with the same base.	9. Give the answer in exponential form: $8^{-4} \div 8^2 =$ ____
10. Adding, subtracting, multiplying, and dividing integers.	10. $-3 + (-7) =$ ____ $-4 \times 7 =$ ____
11. Translating a word phrase to a literal expression.	11. Write this as a literal expression: Five less than the sum of two numbers, a and b. ____________
12. Solving an equation with one variable and two operations.	12. $2a + 40 = 60$; $a =$ ____
13. Finding the value of an open phrase with two variables, given the value of the variables.	13. $a + b + 17 =$ ____ $(a = 12,\ b = 15)$

PRE-ALGEBRA B,C

14. Deducing the two-step rule of a function machine and using the rule to determine further output.

14. Figure out the rule for this function machine, and use it to fill in the blank output spaces.



INPUT	OUTPUT
1	1
2	4
3	7
4	
5	

15. Identifying the missing information needed to solve a word problem that has insufficient information.

15. The sun is 93 million miles from the earth. How long would it take a meteor to travel from the sun to the earth? What additional information do you need to solve this problem?

____

____

16. Solving one-, two-, and three-step word problems involving computation at this level, some including extraneous information.

16. A woman saved $150 a month from her $850-a-month salary. How long would it take her to save enough money to buy five $48 dining room chairs?

____

C. **FRACTIONS**

1. Adding and subtracting common fractions with unlike denominators, giving answers in simplest terms, written horizontally.

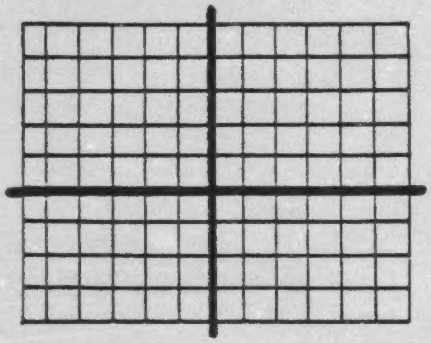
1. $\frac{4}{5} + \frac{3}{8} =$ ____; $\frac{2}{3} - \frac{1}{9} =$ ____

2. Finding a fractional part of a large number, expressed in lowest terms, in word problem form.

2. 350 out of 420 people voted for a candidate in an election. What fraction is this, in simplest terms?

3. Adding and subtracting mixed numbers with unlike denominators, without regrouping, written vertically, giving answers in simplest terms.	3. $\begin{array}{r} 4\frac{1}{3} \\ +9\frac{2}{5} \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6\frac{1}{6} \\ -4\frac{1}{9} \\ \hline \end{array}$
4. Multiplying two or three common fractions, mixed fractions, or whole numbers, including finding a fraction of money, giving simplified answers.	4. $\frac{1}{3}$ x $2\frac{3}{4}$ x 4 = ____; $\frac{2}{3}$ of $51 = ____
5. Dividing common fractions, mixed fractions, or whole numbers, giving answers in simplest terms.	5. $1\frac{1}{5} \div \frac{1}{2}$ = ___; $\frac{4}{5} \div 8$ = ___
6. Identifying numerator, denominator, reciprocal, mixed fraction, and improper fraction without benefit of a list.	6. What is the bottom number in a fraction called? What is a whole number combined with a common fraction called? What is the reciprocal of $\frac{5}{6}$?
7. Solving ratio and proportion problems in fraction and ratio form.	7. 3:12 = 6:____ $\frac{2}{5} = \frac{6}{n}$; n = ___
D. DECIMALS AND PERCENT	
1. Adding and subtracting decimals and mixed decimals with different numbers of decimal places, written horizontally (maintained). Students must stack up numbers correctly.	1. 7.54 + 998.02 + 56 = ______ 5.003 - .9801 = ______
2. Multiplying decimals, mixed decimals, and whole numbers, with some problems written vertically or horizontally, needing one or more zeroes to the left of the product.	2. .164 x .032 = ______
3. Dividing decimals, mixed decimals, and whole numbers with two-digit divisors, some with additional zeroes required in the dividend, rounding answers to a specific place.	3. 65.8 ÷ .0032 = _____ (to the nearest tenth)

E. ESTIMATING, GRAPHS, ETC.

1. Finding the intersection, union, and complement of sets and recognizing disjoint sets, using numerical form or Venn diagrams.	1. What is the union of sets A and B? What is the intersection? A {4, 5, 6, 7, 8...} B {6, 8, 10, 12, 14}
2. Plotting points on a Cartesian plane (maintained).	2. Plot these points on the Cartesian plane and label: A (-3,5); B(3,-4) 

ALGEBRA I OBJECTIVES

Introduction

These objectives for Algebra I have been developed to give students strong, main-line algebraic experience. Upon satisfying these objectives, students will have had the opportunity to experience some of the power as well as the beauty of algebra. Students will also be prepared for more advanced mathematics courses.

Every attempt has been made to keep these objectives consistent with the curriculum found in the major schools and textbooks around the nation. To alter or weaken objectives because of current levels of student attainment in Boston does not seem a fair thing to do - especially for the future success of our students.

These curriculum objectives were developed with help from many sources. Of particular importance was the set of objectives developed by a group of Boston mathematics teachers during the 1981-2 school year. These objectives, entitled Algebra I Curriculum Outline, set the tone and direction for the entire set of objectives that emerged.

The Algebra I Objectives are divided into the following sections:

A. Structure of the Real Number System
B. Algebraic Structure - Variables, Sets, and Phrases
C. Equations and Inequalities
D. Exponents
E. Polynomials
F. Special Products and Factoring
G. Rational Expressions
H. Graphs of Linear Equations and Inequalities
I. Irrational Numbers (Radicals)
J. Relations and Functions
K. Quadratic Equations
L. Algebra I and Careers

ALGEBRA I

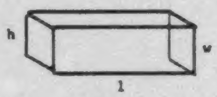
A. STRUCTURE OF THE REAL NUMBER SYSTEM

① Identifying natural numbers, whole numbers, integers, rational numbers, and irrational numbers.	1. $A = \{-5, \frac{1}{2}, -3, -1, 0, 1.4, \sqrt{2}, 2, \pi, \frac{7}{4}, -23.1\}$ Identify the (a) rational numbers (b) integers (c) irrational numbers
② Representing numbers on a number line.	2. Place the numbers $-4, \frac{1}{3}, \frac{5}{4}, 2, -\frac{1}{2}, \pi, 3$, on a number line.
③ Comparing numbers using an inequality or equal symbol.	3. Replace the ? with <, >, or =. $3 \;?\; 7$, $8 \;?\; 6$, $-\frac{14}{3} \;?\; -\frac{13}{3}$, $-17.4 \;?\; -15.8$, $3 \;?\; -5$
④ Arranging numbers in order.	4. List in order from smallest to largest: (a) $\frac{1}{3}, -\frac{3}{4}, \frac{5}{6}, 0, -\frac{2}{3}, .875$ (b) $-7.8, \pi, -\sqrt{5}, 3.2, -8.3, -\sqrt{7}$
⑤ Recognizing equivalent real numbers.	5. Group names for the same number together. (a) $-\frac{6}{3}, \frac{5}{2}, -4, 2.5, -2$ (b) $1.75, \frac{4}{3}, -1.4, 1.333\ldots, \frac{14}{8}$
⑥ Evaluating the absolute value of a number.	6. Give the value of ? (a) $\lvert 7-4 \rvert = ?$ (b) $\lvert 4-7 \rvert = ?$ (c) $-\lvert 3-8 \rvert = ?$ (d) $\lvert -5-2 \rvert = ?$

⑦ Performing the four basic operations with real numbers without the use of a calculator.	7. Evaluate: (a) $3-(8-2) = ?$ (b) $-4.2 \div (-2) = ?$ (c) $(\sqrt{5})\ (\sqrt{5}) - (-3) = ?$ (d) $-\frac{3}{4} + \frac{3}{8} = ?$
⑧ Performing the four basic operations with real numbers using a calculator.	8. Use a calculator to evaluate (a) $\frac{(1.5)(8.2)}{(-3.7)} = ?$ (b) $\frac{376.12}{7} + \frac{485.32}{3} = ?$ (c) $278.23 - (48.9 \div 3) = ?$ (d) $3\sqrt{5} - 7(4.6) = ?$
⑨ Identifying the field properties of the real number system.	9. Name the property or properties used: (a) $3(7+8) = (3\cdot 7) + (3\cdot 8)$ (b) $(-7)(-\frac{1}{7}) = 1$ (c) $(\sqrt{5}\cdot 2)5 = \sqrt{5}(2\cdot 5)$ (d) $0+a = a+0 = a$
B. ALGEBRAIC STRUCTURE - VARIABLES, SETS, AND PHRASES	
① Identifying variables and constant quantities.	1. Identify the variables and the constants in this listing: $3,\ x,\ \pi,\ -5,\ 3a,\ y,\ \frac{5b}{2},\ \sqrt{7}$
② Identifying the following: term, factor, coefficient, exponent.	2. In the following expressions, identify the term or terms, factors, coefficients, and exponents. (a) $(-3a^2) + 72$ (b) $(4b^3)(12b)$ (c) $\frac{25^3}{3} - \sqrt{5}\ s^2 + 3s$

③ Describing the subsets of sets.	3. $A = \{1, 7, \frac{17}{2}\}$ List the subsets of A.
④ Finding the union and the intersection of sets of numbers.	4. $A = \{3,7,12\}$, $B = \{2,8,12,15\}$ $A \cup B = ?$ $A \cap B = ?$
⑤ Constructing a Venn diagram to illustrate operations with sets.	5. $R = \{10,12,15\}$, $S = \{15,17,20,22\}$ Show $A \cap B$ using a Venn diagram
⑥ Translating phrases to mathematical symbols.	6. Use mathematical symbols to translate these phrases: (a) m is more than n (b) p decreased by 7 (c) six times r (d) four times the number k and then increased by 3
⑦ Evaluating numerical and algebraic expressions involving parentheses.	7. Simplify the following: (a) $4(3a-a) = ?$ (b) $[(6b \div 3)+2b]5 = ?$ (c) $6\left(\frac{12r}{9} + \frac{2r}{3}\right) = ?$
⑧ Applying variables in realistic situations.	8. In many schools, for a grade of A in a course, 4 points are given for each credit unit in that course, 3 points for a B, 2 points for a C, 1 point for a D, and 0 points for an F. (a) If a student has a credits with A grades, b credits with B grades, c credits with C grades, d credits with D grades, and e credits with F grades, how many points are earned? (b) What is the grade-point average?

C. EQUATIONS AND INEQUALITIES

① Translating English phrases into algebraic phrases.	1. Give the mathematical equation or inequality for the following English sentences: (a) Three times a number increased by five is equal to twenty-six. (b) Three more than a certain number decreased by five equals ten. (c) Two times a certain number increased by three is more than fifteen.
② Evaluating algebraic phrases when given replacement values.	2. Evaluate the given algebraic phrase, using the replacement values for each variable. (a) if $x = 4$, and $y = -3$, then $2x+4y = ?$ (b) if $r = -7$ and $s = 12$, then $4r-s \geq ?$
③ Evaluating mathematical formulas given the replacement values for the variables.	3. Use the replacement values to evaluate the formula. Volume = length x width x height  If $l = 5$cm, $h = 6$cm, and $w = 4$cm, then $V = ?$
④ Solving an equation with one variable and any combination of the four operations. Give the properties used in the solution.	4. (a) $3(x+1)+2 = -7 \quad x = ?$ (b) $(\frac{1}{2})m - 3 = -1 \quad m = ?$ (c) $-\frac{d}{4} - 2 = -1 \quad d = ?$
⑤ Solving an equation with one variable contained in two or more terms. Give the properties used in the solution.	5. Solve for x and give a mathematical property for each step in the solution process. $5x-8 = 2x+4$ Given $5x-2x-8 = 2x+4-2x$ __________ $3x-8+8 = 4+8$ __________ $\frac{3x}{3} = \frac{12}{3}$ __________ $x = 4$ __________

ALGEBRA I C

⑥ Recognizing the effects of the operations +, −, x, ÷ on inequality sentences.	6. Give the correct inequality symbols for the following problems. (a) $(8)(-2)\underline{?}(3)(-2)$ (b) $\frac{(8)}{(2)}\ \underline{?}\ \frac{(3)}{(2)}$ (c) $\frac{(8)}{(-2)}\ \underline{?}\ \frac{(3)}{(-2)}$ (d) $(8)(2)\underline{?}(3)(2)$ (e) $(8)+(-5)\underline{?}(3)+(-5)$ (f) $(8)-(12)\underline{?}(3)-(12)$
⑦ Solving a linear inequality with one variable. Give the properties used in the solution.	7. Solve for x and give a mathematical property for each step in the solution process. $-2x+5 < -7$ Given $-2x+5-5 < -7-5$ ______ $\frac{-2x}{-2} > \frac{-12}{-2}$ ______ $x > 6$ ______
⑧ Solving word problems, using linear equations and linear inequalities with one variable.	8. Using linear equations and inequalities, find the length and width of a rectangle if the length of the rectangle is 5 cm more than the width, and if the perimeter of the rectangle is 42 cm.
9. Graphing the solution set of a linear equation on the number line.	9. Solve the linear equation $2(x-3) = 3x-10$ and graph the solution on the number line.
10. Graphing the solution set of a linear inequality on the number line.	10. Solve the linear inequality $-3x+5 < -7$ and graph the solution set on the number line.

11. Applying linear equations and linear inequalities in realistic situations.	11. Your class wants to sponsor a school dance. One band will play for \$250 plus 50% of the gate and another band will play for the flat rate of \$550. (a) If you expect about 300 people to attend, what is the most you can charge each person and still make the first band your best buy? (b) If you charge this maximum rate, about how much money will be left to spend on other expenses for the dance?
12. Checking the solution set for a linear equation.	12. Which of the following values of x are in the solution set of $3x-2 = 4x-6$? $x = -3$, $x = 4$, $x = 5$
13. Checking the solution set for a linear inequality.	13. If $3x-12 < -10+4x$, which of the following x values are in the solution set? $x < 5$, $x > 7$, $x < -4$
14. Using a calculator to check the solution set for a linear equation and/or a linear inequality.	14. Given the linear equation, $4.5x + 3 = 3.1x + 6.5$, use a calculator to find which of the following values of x are in the solution set. $x = -2.5$, $x = 25$, $x = 2.5$
D. EXPONENTS	
① Identifying the base and the exponents in any term.	1. Name the base and the exponent(s) for each of the following: (a) $(-7)^2$ (b) $5a^2c^3$ (c) $(3b)^2$
② Evaluating terms involving exponents,	2. Evaluate the following if $a = 3$, $c = -1$. (a) $4a^2c^3$ (b) $\frac{(-3a)^2}{c^2}$ (c) $(5a^2 \div 2c^2)^3$

③ Rewriting numbers in the simplest exponential form, using prime factors.	3. Write as a product of prime factors: (a) 8,108 (b) 12,000
④ Raising an exponential expression to a power.	4. Carry out the indicated operations: (a) $(-5x^3y^2)^2$ (b) $[(3r^2)r]^3$
⑤ Multiplying expressions involving exponents.	5. Carry out the indicated operations: (a) $(-3x^2y)(5x^3y)$ (b) $(4m)^2(-2m^3n)$
⑥ Dividing expressions involving exponents.	6. Carry out the indicated operations: (a) $\frac{-27x^5y^2}{-3x^2y^2}$ (b) $(-5m)^2n^3 \div 3m^3n$
⑦ Using exponents in realistic situations.	7. The collision impact I (kinetic energy) of an automobile with mass m and speed s is given by the formula $I = Kms^2$. If the speed is tripled, what happens to the collision impact of a 1000 kg. car?
8. Using a calculator to evaluate expressions containing exponents.	8. Use a calculator to find the results for the following: (a) $(8.74)^3 = ?$ (b) $(\frac{6.4}{3})^2 = ?$ (c) $\frac{(6.9)^2+(2.7)^2}{(3.8)^2 - 2.9} = ?$
E. POLYNOMIALS ① Identifying monomials, binomials, trinomials, and polynomials.	1. Classify the following as monomials, binomials, trinomials, or polynomials. (a) $5x-2y$ (b) $3x$ (c) $9x^3-5x^2+x-7$ (d) $7y^2+5y-8$ (e) $(3x^2)(2x)$

ALGEBRA I E,F

② Identifying the degrees of polynomials.	2. Give the degrees of the following polynomials. (a) $4x^2 - 7$ (b) $3x^3y + 5x^2 - 12$ (c) $4mn^2 - 3m^2n^3 - 5mn$ (d) $7rs^3t^2 - 5r^2s^2t^3$
③ Simplifying polynomials by combining similar terms.	3. Simplify the following expressions: (a) $8x^2-7x+3-8x+2$ (b) $4a+4b-3a^2-b+5a^2-2b^2$ (c) $3m^2-12m+5+3m-8$
④ Adding and subtracting polynomials.	4. Perform the indicated operations: (a) $(5x-7)+(-2x-10) = ?$ (b) $(9x^2-6x+8)-(2x^2-6x+2) = ?$
⑤ Multiplying polynomials.	5. Perform the indicated operations: (a) $-2x(3x^2-7x+8) = ?$ (b) $(5x-2)(3x+7) = ?$ (c) $(3a-4)(5a^2+7a-2) = ?$
⑥ Dividing polynomials.	6. Perform the indicated operations: (a) $\frac{a^3b+a^2b^2-5ab^3}{ab}$ (b) $(x^2-9x+20) \div (x-4)$
⑦ Using polynomials in realistic situations.	7. Suppose you work for $3.50 an hour. Of the total you earn, 25% is deducted for income tax, social security, and union dues. How many hours must you work in a week to take home $140?

F. SPECIAL PRODUCTS AND FACTORING

① Factoring positive integers.	1. Write the prime factorization of (a) 224 (b) 196
② Finding the greatest common factor of two numbers.	2. Write the greatest common factor of (a) 196 and 224 (b) $32a^3b^2$ and $12a^2b$

ALGEBRA I F

③ Identifying and factoring a monomial factor in a polynomial.	3. Factor: (a) $18x^3y-48x^2y+9xy$ (b) $36m^2n^2-24mn^3-30n^4$
④ Identifying and factoring special types of polynomials (difference of two squares).	4. Factor the following: (a) $4a^2-9b^2$ (b) t^2-16s^2 (c) p^4-q^4 (d) x^4-1
⑤ Identifying and factoring special types of polynomials (perfect square trinomials).	5. Factor the following: (a) a^2+6a+9 (b) $4r^2+12rs+9s^2$ (c) $16p^2-24pq+9q^2$
⑥ Identifying and factoring general trinomials.	6. Factor the following: (a) $x^2+7x+12$ (b) $x^2-9x+20$ (c) x^2+x-20 (d) $x^2-2x-35$ (e) $2x^2-xy-15y^2$
⑦ Checking the factoring of polynomials by multiplication.	7. Factor the following polynomials and check your result by multiplication of the factors. (a) $4x^2-4x-8$ (b) $2r^2+r-6$ (c) $6x^2+5x-6$
⑧ Combining several types of factoring.	8. Factor the following polynomials: (a) $a^2+b^2+2ab-4c^2$ (b) $-16e^2f-10e^2fg-e^2fg^2$
⑨ Identifying prime polynomials.	9. Identify the prime polynomials in the following list: (a) a^2-4b^2 (b) a^2+9b^2 (c) x^2+x+3 (d) x^2+x-2 (e) x^2+5x+9 (f) $7x^2+9$
⑩ Solving polynomial equations by factoring.	10. Solve for x: (a) $x^2-7x = 18$ (b) $x^2+4x = 0$ (c) $2x^2+20x = -50$

⑪ Solving application problems by factoring.	11. The length of a rectangle is 5 meters more than the width. The area of the rectangle is 84 square meters. Find the dimensions of the rectangle.

G. RATIONAL EXPRESSIONS

① Identifying rational expressions.	1. Identify the rational expressions in the following: $4a+3b, \frac{3c}{4}, (7d-2)e, \frac{(a^2-b^2)}{4a},$ $15s^2 \div 4t, (4a)(3b^2)$
② Identifying equivalent rational expressions.	2. Identify the equivalent rational expressions in the following: $\frac{3d^2}{4e}, \frac{a-b}{3}, \frac{a^2+b^2}{3c}, \frac{a^2-b^2}{3(a+b)},$ $\frac{2(a^2+b^2)}{6c}, \frac{4a-4b}{12}, \frac{a+b}{c}, \frac{9d^3}{12de}$
③ Identifying the values of variables for which a given rational expression is undefined.	3. Give the restrictions for the variables in each of these rational expressions. (a) $\frac{3a}{b-2}$ (b) $\frac{4c^2+b^2}{d^2-1}$ (c) $\frac{2m^2+n}{2(m-3)}$ (d) $\frac{5s^2}{3rt}$
④ Writing rational expressions in the simplest form.	4. Write in the simplest form: (a) $\frac{18}{48}$ (b) $\frac{5a^2b^3}{25ab^2}$ (c) $\frac{a^2-4b^2}{a^2-4ab+4b^2}$
⑤ Finding the lowest common denominator of fractions with monomial denominators.	5. Find the lowest common denominator of the following: (a) $\frac{1}{3}, \frac{5}{6}, \frac{11}{18}, \frac{3}{5}$ (b) $\frac{5}{7a^2}, \frac{4}{21b^2}, \frac{3}{4ab}$
⑥ Adding and subtracting fractions with monomial denominators.	6. Combine the following: (a) $\frac{4x+3}{4} + \frac{2x-1}{9} + \frac{7x+5}{8}$ (b) $\frac{5x+2}{6y^2} - \frac{4x-3}{15y}$

ALGEBRA I G

⑦ Multiplying fractions with monomial denominators.	7. Perform the indicated operations: (a) $\frac{4x}{5y^2} \cdot \frac{3y}{7x^3}$ (b) $\frac{3r^3s}{7t} \cdot \frac{14t^2}{9rs^2}$
⑧ Identifying reciprocals (inverses).	8. Write the multiplicative inverse: (a) $\frac{8}{5}$ (b) $\frac{-7}{11}$ (c) $\frac{9}{13}$ (d) $\frac{3}{5a}$
⑨ Dividing fractions with monomial numerators and denominators.	9. Perform the indicated operations: (a) $\frac{3}{8} \div \frac{9}{16}$ (b) $\frac{4x^2}{5y} \div \frac{12x^3}{15y^2}$
⑩ Using multiplication of (-1) to change signs of rational expressions.	10. Simplify by changing signs: (a) $-(-\frac{5}{8})$ (b) $\frac{x-5}{5-x}$
⑪ Finding the lowest common denominator of fractions with polynomial denominators.	11. Find the lowest common denominator of the following: (a) $\frac{4}{x-2}, \frac{7}{3x+6}, \frac{5}{x^2-4}$ (b) $\frac{x}{4x^2-100}, \frac{2}{x+5}, \frac{-3}{5-x}$
⑫ Adding and subtracting rational expressions with polynomial denominators.	12. Perform the indicated operations: (a) $\frac{2}{a+b} + \frac{5}{a^2-b^2} + \frac{7}{2a+2b}$ (b) $\frac{6}{x-y} - \frac{4}{x+y}$
⑬ Multiplying and dividing rational expressions containing polynomials.	13. Perform the indicated operations: (a) $\frac{2x-6}{x+3} \cdot \frac{x^2+3x}{3x-9}$ (b) $\frac{x+3}{x+4} \div \frac{4x+12}{x^2-16}$
⑭ Using the least common multiple to simplify complex fractions.	14. Simplify: (a) $\frac{8\frac{1}{2}}{9\frac{2}{3}}$ (b) $\frac{\frac{3}{x}}{\frac{7x}{y}}$

ALGEBRA I G

15. Evaluating rational expressions using a calculator.	15. Use a calculator to evaluate the following: (a) $\frac{7}{12} + \frac{9}{16}$ (b) $\frac{5}{8} - \frac{11}{20}$ (c) $\frac{17}{40} \cdot \frac{19}{50}$
16. Solving open sentences containing rational expressions.	16. Solve for x: (a) $\frac{3}{x} = \frac{4}{5}$ $(x \neq 0)$ (b) $\frac{21}{x+1} = 7$ $(x \neq -1)$
17. Solving fractional equations and checking answers.	17. Solve and check your answers: (a) $\frac{x+1}{2x-2} = \frac{x}{6} + \frac{1}{x-1}$ (b) $\frac{y-2}{y} - \frac{y-3}{y-6} = \frac{1}{y}$
18. Solving inequalities containing rational expressions and checking answers.	18. Solve and check your answers: (a) $\frac{3}{2}p - 5 < 4$ (b) $-2 \leq 2 - \frac{k}{3}$
19. Applying rational expressions in solving word problems.	19. It takes Bill 5 hours to do a job and it takes Jack x hours to do the same job. Working together, they finish the job in 3 hours. How long will it take Jack to do the job alone?
20. Solving problems involving ratios.	20. A profit of $1800 is to be divided among 3 people in the ratio of 8:3:4. How much should each person receive?
21. Solving problems involving proportions.	21. A truck can pump 8001 of oil in 25 minutes. How much oil can it pump in 2 hours?

H. GRAPHS OF LINEAR EQUATIONS AND INEQUALITIES

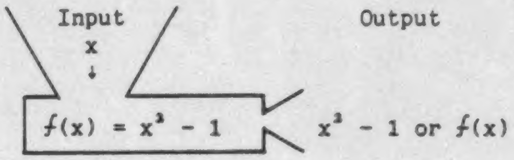
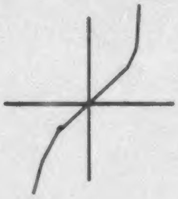
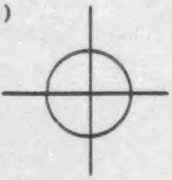
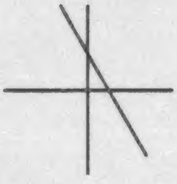
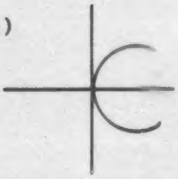
① Locating points and naming quadrants on a coordinate system.	1. Name the quadrant in which each ordered pair lies and then plot the point on the Cartesian coordinate system. (2,3), (-3,4), (4,-2), (-4,-5), (0,0), (-5,0)
② Listing ordered pairs that satisfy a given linear equation.	2. Identify 4 sets of ordered pairs that satisfy each of these linear equations. (a) $3x-2y = 12$ (b) $5x = 4y+5$ (c) $7y = 11-6x$
③ Finding the slope of a line given two points.	3. Find the slope of $\overleftrightarrow{AB}$ if (a) A is (4,5) and B is (6,8) (b) A is (-3,7) and B is (-2,-3)
④ Writing the equation of a line given the slope and one point.	4. Write the equation of the line with slope $m = -\frac{3}{4}$ and containing the point C (-2,4).
⑤ Writing the equation of a line given the slope and the y-axis intercept.	5. Write the equation of the line with slope $m = \frac{4}{5}$ and crossing the y-axis at B (0,7).
⑥ Writing the equation of a line given two points on the line.	6. Write the equation of a line passing through the points A (6,4) and B (-10,5).
⑦ Graphing the line $Ax+By = C$ by plotting points.	7. Graph the lines by plotting points: (a) $3x-4y = 12$ (b) $(-\frac{2}{3})x+5y = -10$

⑧ Graphing a line given a point and the slope.	8. Graph the line containing P (3,5) with slope $m = -\frac{2}{3}$.
⑨ Graphing a linear equation using the slope-intercept method.	9. Graph the line $\frac{x}{4} + - \frac{y}{6} = 1$ by using the slope-intercept method.
⑩ Solving a system of equations using addition or subtraction.	10. Solve for (x,y) using the addition or subtraction method: (a) $2x+3y = 7$ $3x-5y = 1$ (b) $2p-3q = 5$ $8 = 3p-3q$
⑪ Solving a system of equations using substitution.	11. Solve for (x,y) using the substitution method. (a) $4x+3y = 22$ $x-4y = -4$ (b) $4a-b = 5$ $3a-2b = -6$
⑫ Graphing a system of equations on one set of axes and giving solutions from the graph.	12. Solve for (x,y) by graphing the two linear equations. (a) $2x+y = -2$ $x-y = -4$ (b) $3x+y = -6$ $2y-3 = -6x$
⑬ Indicating whether the linear system is inconsistent, consistent, or dependent.	13. Tell whether these linear systems have no solutions, an infinite set of solutions, or just one solution. (a) $x-y = -2$ $6+3x = 3y$ (b) $2x-y = 1$ $3x+y = -6$
⑭ Locating the region on a coordinate system for the solution of $Ax + By > C$.	14. Graph the solution region for (a) $4x - 7y < 28$ (b) $3x + 5y > -12$
⑮ Solving a system of inequalities by graphing.	15. Solve this system of inequalities by graphing: $x + 2y > 5$ $2x - y \le 8$

⑯ Solving geometric word problems involving two linear equations.	16. The perimeter of a rectangular field is 100m. Find the dimensions, if the width is two-thirds the length.
⑰ Solving application problems involving two linear equations.	17. Flying against an 80km/h head wind, a plane can fly a certain distance in four hours. Flying in the opposite direction with the same wind blowing as a tail wind, it can fly that distance in 1 hour less. What is the plane's air speed?
⑱ Solving number theory problems involving two linear equations.	18. The sum of the digits in a two-digit number is 12. The new number obtained when the digits are reversed is 24 less than three times the original number. Find the original two-digit number.
I. IRRATIONAL NUMBERS (RADICALS)	
① Finding the roots of numbers that have rational roots.	1. Find the following roots: $\sqrt{64}$, $\sqrt[3]{-27}$, $\sqrt[4]{625}$, $\sqrt{169}$
② Approximating the square root of a number that is not a perfect square.	2. Without using tables or a calculator, find an approximate value for $\sqrt{32}$, $\sqrt{8}$, $\sqrt{70}$
3. Finding the root of a number, using a calculator.	3. Use a calculator to find $\sqrt{17}$, $\sqrt[4]{35}$, $\sqrt[3]{-50}$, $\sqrt{600}$
④ Simplifying radicals with numerical radicands.	4. Give in simplified radical form: $\sqrt{192}$, $\sqrt[3]{54}$, $\sqrt{1352}$

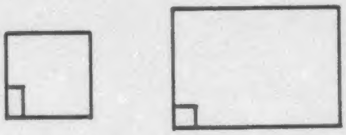
ALGEBRA I I

⑤ Simplifying radicals with radicands containing algebraic expressions. (All variables represent positive numbers.)	5. Give in simplified radical form: $\sqrt{8a^3b^4}$, $\sqrt[3]{256x^9y^6}$
⑥ Multiplying radicals with the same indices.	6. Perform the operations, leaving answers in simplest radical form. (a) $8\sqrt{3} \cdot 4\sqrt{12}$ (b) $8\sqrt{a^5} \cdot 5b\sqrt{ab^3}$ (c) $5\sqrt[3]{54} \cdot 2\sqrt[3]{128}$
⑦ Multiplying radicals with different indices.	7. Perform the operations, leaving answers in simplest radical form. (a) $4\sqrt[3]{2} \cdot 6\sqrt{3}$ (b) $(5\sqrt{x})(-3\sqrt[3]{x^2})$
⑧ Combining radicals by addition and/or subtraction.	8. Perform the operations, leaving answers in simplest radical form. (a) $4\sqrt{27} + 2\sqrt{3} + 5\sqrt{12}$ (b) $12\sqrt{18} - 5\sqrt{8}$
⑨ Rationalizing expressions with radical denominators.	9. Rationalize the denominator: (a) $\frac{6}{\sqrt{2}}$ (b) $\frac{8}{\sqrt{3} - 2}$
⑩ Solving and checking equations with radical expressions.	10. Solve and check these equations: (a) $\sqrt{x-1} - x + 7 = 0$ (b) $\sqrt{5x^2-36} - x = 0$
11. Solving geometric problems involving radicals. ⑫ Solving application problems involving radicals.	11. Find the length of the diagonal of a square with a side of 3 cm. 12. If the square root of a number is decreased by 2, the result is 3. Find the number.

J. RELATIONS AND FUNCTIONS	
① Identifying the domain and range of sets of several ordered pairs.	1. List the domain and range of these sets of ordered pairs: (a) {(3,4), (4,6), (5,8), (6,10)} (b) {(5,11), (7,15), (3,7), (2,5)}
② Drawing a graph representing a relation.	2. Graph these relations: (a) $y = 5x - 2$ for $x \varepsilon \{1,2,3,4,5\}$ (b) $y = \pm\sqrt{100-x^2}$ for $x \varepsilon \{6,8,10\}$
3. Defining functions in terms of a mapping (rule).	3.  Input x $f(x) = x^2 - 1$ Output $x^2 - 1$ or $f(x)$ This function machine is for the function f that assigns to each input x the output $x^2 - 1$. Find $f(3)$, $f(8)$, and $f(-6)$.
④ Recognizing that a function value can be designated by y or $f(x)$.	4. Rewrite these function rules in terms of y. (a) $f(x) = 3x - 2$ (b) $f(x) = 18 - x$ (c) $f(x) = (3x)^2 - 7$
⑤ Determining if a relation is a function, using the vertical line test.	5. Which diagrams represent functions and which represent relations that are not functions? (a)  (b)  (c)  (d) 

⑥ Graphing a linear function.	6. Graph these linear functions. (a) $f(x) = 2x + 2$ (b) $f(x) = -3x - 5$ (c) $f(x) = \frac{3}{4}x - 3$
7. Finding the range of a known function for a given domain.	7. Find the range of $f(x) = 6x+2$ if the Domain = $\{x: -2 \leq x \leq 4\}$ Range =
8. Finding the inverse of a given linear function.	8. Find the inverse function for (a) $f(x) = 3x$ (b) $f(x) = 5x - 3$
9. Graphing the inverse of a given linear function.	9. Graph $y = 2x + 4$ and its inverse function $y = \frac{x}{2} - 2$ on the same coordinate system.
K. QUADRATIC EQUATIONS	
① Identifying quadratic equations and their component terms.	1. Identify the quadratic equations from the following. For each quadratic equation, identify the quadratic term, the linear term, and the constant term. (a) $7x^2 + 9 = 0$ (b) $y = 3x - 5$ (c) $4x^2 - 3x = 8$ (d) $5x^3 - 2x^2 - 9 = 0$ (e) $13x - 11 = 6x^2$
② Solving quadratic equations by factoring.	2. Solve for x by factoring: (a) $x^2 - x - 42 = 0$ (b) $6x^2 - 5x - 4 = 0$ (c) $4x^2 - 10x + 6 = 0$
③ Solving quadratic equations by completing the square.	3. Solve for x by completing the square: (a) $x^2 + 14x + 24 = 0$ (b) $3x^2 + 5x - 1 = 0$ (c) $x^2 + 8x = -4$

ALGEBRA I K

④ Solving quadratic equations by using the quadratic formula.	4. Solve for x, using the quadratic formula: (a) $x^2 + 3x - 18 = 0$ (b) $5x^2 + x - 3 = 0$
5. Solving quadratic equations by using the quadratic formula and a calculator.	5. Solve for x, using the quadratic formula and a calculator: $x^2 + 8x + 15 = 0$
⑥ Identifying roots of quadratic equations as real or not real, and rational or irrational if real.	6. Describe the nature of the roots of these quadratic equations, using the discriminant: (a) $x^2 + 5x + 1 = 0$ (b) $x^2 + 4x + 5 = 0$ (c) $3x^2 + 5x - 2 = 0$
7. Graphing quadratic equations.	7. Graph, on the Cartesian plane, (a) $y = x^2 + 6x + 9$ (b) $y = x^2 - 5x - 6$
8. Solving a quadratic inequality on the number line.	8. Solve and check $x^2 - 4x - 12 \leq 0$ and show the solution on the number line.
9. Graphing a quadratic inequality on the Cartesian coordinate system.	9. Graph these quadratic inequalities on the Cartesian plane: (a) $y \geq x^2 - 2x + 1$ (b) $y \leq x^2 - 2x - 3$
10 Solving geometric problems, using quadratic equations.	10. Given a square and a rectangle with the dimensions shown in the figures, the length of a side of the square is x cm. Find the dimensions of both the rectangle and the square, if their combined area is 765 sq. cm. 

11. Solving problems involving distance and time, using quadratic equations.	11. An object is thrown straight up with an initial velocity of 20 meters per second. After t seconds, its distance above the ground is $s = 20t-5t^2$ meters. Tabulate the values of s for t = 0,1,2,3, etc. What is the maximum height the object reaches?

L. ALGEBRA I AND CAREERS

Investigating one job from each of three occupational clusters in which the skills learned in Algebra I can be applied.

GEOMETRY OBJECTIVES

Introduction

The geometry course suggested by this list of objectives has three main emphases. The first is visual thinking; an attempt has been made, for every area of material in the course, to provide some objectives that stress visual thinking and visual demonstrations (the exercises for such objectives typically start with "Draw..." or "Identify..."). The second emphasis is problem-solving; an attempt has been made to provide an array of problems involving numerical calculation and algebra (these objectives begin with "Find..." or "Calculate..."). The third emphasis is logical structure and proof; for each area of the material, there are assertions to be proved.

In dealing with all three emphases, an attempt has also been made to provide problems that involve both newly learned concepts and the full range of each student's mathematical background. For example, at the end of Unit E on triangles, there is a set of five problems that draw on earlier geometry and on algebra, and from then on through the rest of the units there is a continuing emphasis on cumulative knowledge and integration of skills.

Some comments on individual units are in order. Unit A is largely review, and will certainly be so for students who have mastered Boston's level eight mathematics objectives. In Unit A, any of the underlying axioms of arithmetic that are to be used will be studied. Just how rigorous these are is left to the individual teacher.

In Unit B and Unit C, Euclid's postulates are introduced. The objectives do not dictate the degree of logical rigor to be used on a daily basis in the classroom. In fact, the objectives in these two units do not even take notice of whether a transformational approach is being used. Transformations - and, for that matter, logical proof - as an aid to understanding geometry, are not prominent in these objectives. It is where the transformations or the logic gets the students that matters, and it is that "where" that the objectives treat. It will be up to the individual teacher, as a matter of pedagogy, to decide how formal an approach to take.

If a first semester cutoff point is desired, the end of Unit G (quadrilaterals) is suggested. Other options are available. Since Units F, G, and H are independent of each other, any one of these units may serve as a final unit for the semester. It should also be noted that the units outlined do not necessarily represent the best teaching order for the material. Units can be taught in some other order, and material within units can be moved to other units. The material in Units G through J is particularly flexible.

Unit M concentrates on transformations themselves and includes some formal work designed to promote an understanding of the geometric material of the earlier units. The main thrust of this particular unit is on the relationships among transformations and on the characterization of geometry as the study of properties left invariant under various transformation. (Note: transformations are not geometry.)

Unit N (logic) has few objectives, although its vocabulary list is extensive. The depth of study of this unit is determined by the teacher. Geometry, it must be asserted over and over, is not a logic course. The proofs are not the point. The problems are the point, and it is hoped that the problems presented here are interesting enough to help make this argument convincing.

Teachers are naturally reluctant to leave logical loopholes; holes in the walls of Euclid's edifice through which a breeze might enter and blow out the Olympic torch now in our hands. That torch burns brightly at higher levels - more brightly than when Euclid lit it, in fact. High school students should certainly work hard on the logical organization of geometry materials, and mastering the logical organization of small bodies of material will certainly help the student to remember it. But it is not necessary at this stage to insist on a rigorous proof for every theorem. There is simply not enough time in one year to take that approach.

The question of time brings up a final point, namely, that all students should study all of units in the course. Every attempt has been made to include objectives that will make every unit accessible to all students. It is not assumed that slower students will cover merely the first few units. All the basic material should be studied by all students. (The circled priority objectives are a guide to a comprehensive basic geometry course, with uncircled items serving as enrichment.) Some students will proceed at the visual and computational level. Others will be comfortable with the proofs. But no student should miss the visual and problem-solving work, and even the brightest and most able students should have the chance to strengthen their visual thinking.

These geometry objectives and problems are designed to offer a challenge to all Boston geometry teachers. It is our hope that this challenge will be an exciting one.

The Geometry objectives are divided into the following sections:

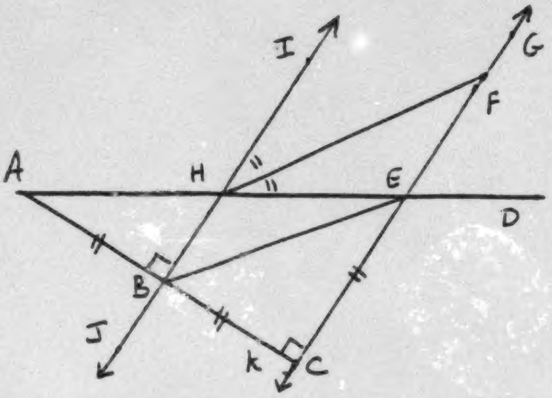
A. Introduction: Review, Basics, Visualization, History
B. Early Structure: Lines, Planes, Angles
C. Triangles, Congruence
D. Perpendicular and Parallel Lines in the Plane
E. Triangles Revisited
F. Perpendicular and Parallel Lines and Planes
G. Quadrilaterals, Regular Polygons
H. Similarity
I. Area
J. The Pythagorean Theorem
K. Circles, Spheres
L. Solids
M. Transformations
N. Logic
O. Geometric Constructions
P. Geometry and Careers
Geometry Course Vocabulary

GEOMETRY

A. INTRODUCTION: REVIEW, BASICS, VISUALIZATION, HISTORY

(1.) Identifying and naming, using the appropriate symbols, the basic terms concerning points, lines, rays, segments, and angles.

1.



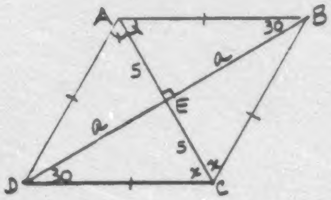
In this figure, find and name examples of

a) acute angles
b) adjacent angles
c) angle bisectors
d) points between pairs of labeled points
e) collinear points
f) complementary angles
g) concurrent lines
h) congruent segments
i) congruent angles
j) endpoints of segments and rays
k) points equidistant from two points
l) points in the interior or exterior of a given angle
m) intersections of lines and segments
n) linear pairs
o) midpoints
p) obtuse angles
q) opposite rays
r) perpendicular lines and segments
s) rays
t) reflex angles
u) right angles
v) segments
w) the sides of a given angle
x) supplementary angles
y) the vertex of a given angle
z) vertical angles

GEOMETRY A

② Using markings in diagrams to identify congruent angles, angle bisectors, right angles, perpendicular segments, congruent segments, and midpoints

2. A2



From the markings in the figure above, find

- a) 3 pairs of congruent angles
- b) 3 sets of congruent segments
- c) an angle bisector
- d) a right angle
- e) a pair of perpendicular segments
- f) the midpoint of any segment
- g) 3 points equidistant from A and C
- h) 2 bisectors of $\overline{DB}$

---

③ Drawing a sketch illustrating given information concerning any of the terms in item (1), above.

3. Draw a sketch to illustrate the following information:

Points A, B and C are distinct and collinear. A is on $\overrightarrow{CB}$ but is not on $\overrightarrow{BC}$. Point D is located such that $\overline{AD} \perp \overline{BD}$. What kind of angle is ADC? (Answer: obtuse.)

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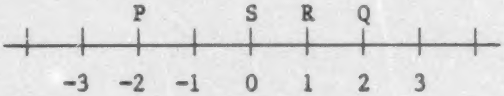
④ Finding missing distances and coordinates, given a line and coordinate system.

4.



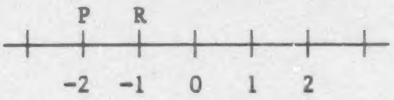
Find:

- a) PQ
- b) SP
- c) QT, if T is the midpoint of $\overline{SR}$
- d) the coordinate of M, if M is on the line and RM = $\frac{1}{2}$ (There are two answers.)

---

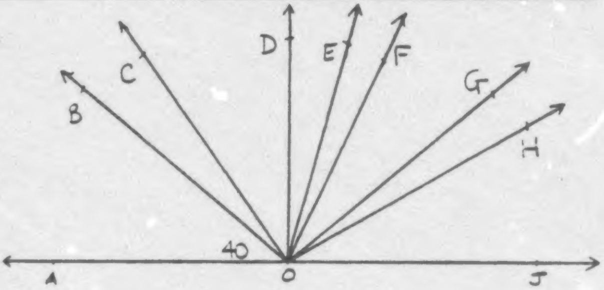
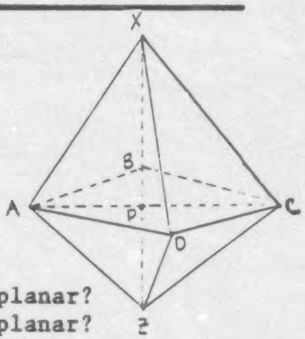
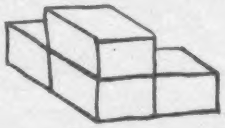
⑤ Finding missing distances and coordinates, using one's mathematics background.

5.



Given the points P and R, find all possible coordinates on the given line for point X if PX = 2RX.

GEOMETRY A

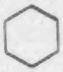
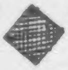
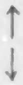
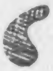
⑥ Finding missing measures of adjacent angles using the addition and subtraction of angles.	6.  Given $m\angle AOB = 40$, $m\angle AOD = 90$, $m\angle COE = 40$, $m\angle DOF = 25$, $m\angle EOF = 10$, find $m\angle BOC$.
⑦ Finding the measure of an angle, given information about its complement or supplement or both.	7. The measure of a certain angle is 2 more than 3 times the measure of its complement. Find the measure of the angle.
8. Reading a two-dimensional drawing of a three-dimensional object to identify coplanar points or collinear lines.	8. In this three-dimensional figure, $\overline{XZ}$ and $\overline{AC}$ intersect at P, and A, B, C, and D are coplanar. a) Does $\overline{BZ}$ intersect $\overline{AZ}$? b) Does $\overline{XD}$ intersect $\overline{BC}$? c) Is B on $\overline{XP}$? d) Are X, B, A, and Z coplanar? e) Are A, Z, C, and P coplanar? 
9. Given a drawing of a simple pile of blocks, redrawing it from another point of view.	9.  This is a pile of 4 blocks all the same shape and size. What would the pile of blocks look like from the top?
10. Knowing who Euclid was, when and where he lived, and what he wrote.	10. The famous textbook which set forth what was then known about geometry in a systematic manner for the first time is called, ______________________, written by ___________ in about ______ B.C. This book was so well prepared that it was used with very few changes until the beginning of our century, and still provides the skeleton for most of the geometry texts used today.

GEOMETRY A,B

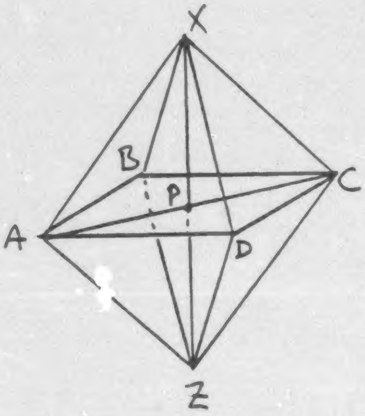
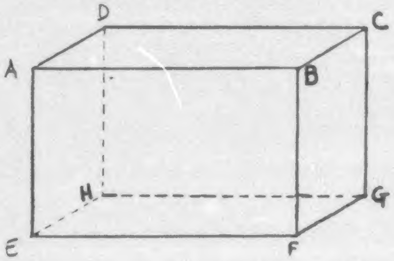
VOCABULARY

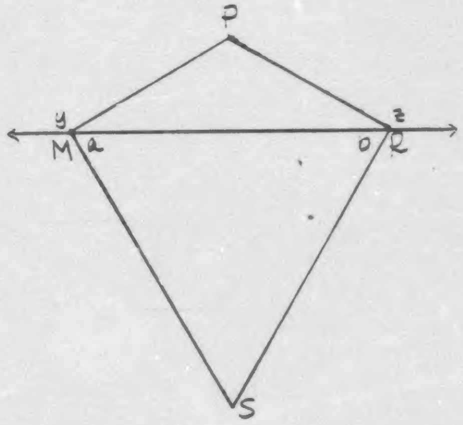
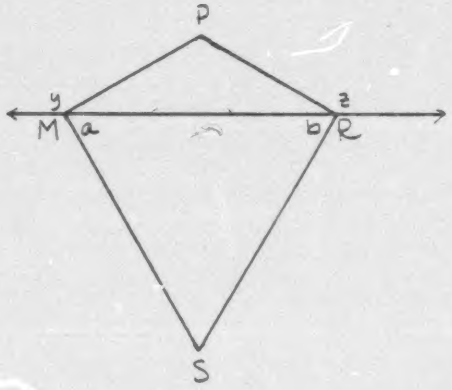
ACUTE ANGLE
ADJACENT ANGLES
ANGLE
ANGLE BISECTOR
BETWEEN
BISECT
BISECTOR
COLLINEAR
COMPLEMENTARY ANGLES
COMPLEMENT OF AN ANGLE
CONCURRENT
CONGRUENT ANGLES
CONGRUENT SEGMENTS
COORDINATE
COORDINATE SYSTEM
COPLANAR
DEGREE
DIMENSION
DISTANCE
DISTINCT
ENDPOINT
EQUIDISTANT
EXTERIOR OF AN ANGLE
INTERIOR OF AN ANGLE
LINE
LINEAR PAIR OF ANGLES
MEASURE OF AN ANGLE
MIDPOINT
OBTUSE ANGLE
OPPOSITE RAYS
PERPENDICULAR LINES
PROTRACTOR
RAY
REFLEX ANGLE
RIGHT ANGLE
SEGMENT
SIDES OF AN ANGLE
SPACE
SUPPLEMENTARY ANGLES
SUPPLEMENT OF AN ANGLE
VERTEX OF AN ANGLE
VERTICAL ANGLES

B. EARLY STRUCTURE: LINES, PLANES, ANGLES

1. Recognizing and understanding convex and nonconvex sets of points.	1. a) Identify each figure as convex or nonconvex; add just enough to the drawing so that it becomes a convex figure. a)  b)  c) . d)  e)  b) Is a triangle a convex set?
2. Finding all the possible results when intersections and unions of convex and nonconvex sets are formed.	2. Let A and B be convex sets, and let X and Y be nonconvex. Decide whether the following sets must be, might be, or cannot be convex. Illustrate with sketches. a) $A \cap B$ b) $A \cap Y$ c) $X \cap Y$ d) $A \cup B$ e) $A \cup Y$ f) $X \cup Y$
3. Demonstrating that a point separates a line into 2 convex half-lines, that a line separates a plane, and that a plane separates space.	3. In your classroom, show where a plane would lie in order to create 2 rooms with half the students in each.

GEOMETRY B

4. Knowing the ways that lines and planes can intersect, and the ways that planes are determined.	4. (a) Line 1 intersects plane E in the distinct points P and Q. What can we say about line 1? (b) Can 4 points be arranged so that they determine - exactly one plane? - exactly 2 planes? - exactly 3 planes? - exactly 4 planes? - no one plane at all?
5. Writing short proofs of basic theorems about the intersection of lines and planes and the determination of planes by specified sets of points.	5. In this 3-dimensional figure, XZ and AC intersect at P; and A, B, C, and D are coplanar. Prove that $\overline{XD}$ does not intersect $\overleftrightarrow{BC}$. 
6. In real space, visualizing planes determined by given sets of points and the intersection of such planes.	6. Given a room whose corners are lettered as shown here, show with your hands the planes determined by these points: a) A, B, G b) A, D, G c) A, F, C  Where do the planes in (a) and (b) intersect? the planes in (b) and (c)?

⑦ Calculating angle measurements by using fundamental information on congruent angles.	7.  In this figure, m ∠ a = 60, ∠ y ≅ ∠ z, and ∠ SMP and ∠ SRP are right angles. What is m ∠ b?
⑧ Writing short proofs involving postulates and theorems of congruent angles.	8.  Given < y ≅ < z, < SMP and < SRP are right angles, prove < a ≅ < b.

VOCABULARY

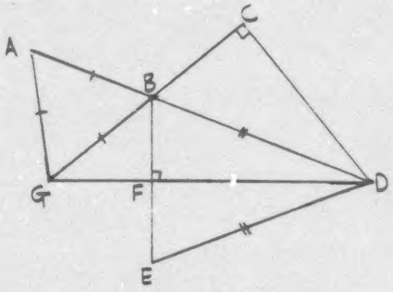
CONVEX SET
DETERMINE
EDGE OF A HALF-PLANE
FACE OF A HALF-SPACE
HALF-PLANE
HALF-SPACE
SEPARATE

GEOMETRY C

C. TRIANGLES, CONGRUENCE

① In a given diagram and in the real world, finding examples of scalene triangle, isosceles triangle (base, base angles, vertex, vertex angle), equilateral triangle, right triangle (hypotenuse, leg), exterior of a triangle, and remote interior angles.

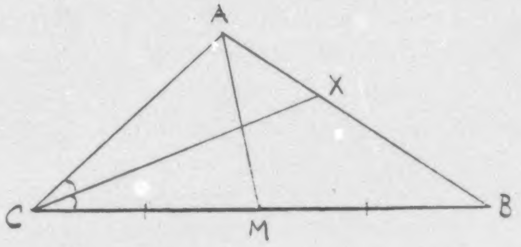
1.



In this figure
- a) describe Δ GBD
- b) find 5 right triangles; and name the hypotenuse of each
- c) describe Δ ABG
- d) find an isosceles triangle that is not equilateral and name its base angles and vertex angle
- e) name an exterior angle for Δ BCD and name the two remote interior angles associated with it

② Recognizing, and distinguishing between, a median and an angle bisector of a triangle.

2.



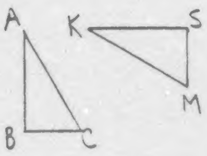
a) In the figure, CX is a(n) ____________ and AM is a(n) _______________.

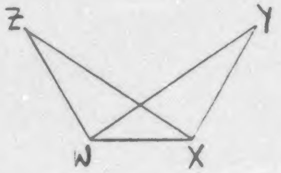
b) A median divides a(n) ___________ of a triangle into two equal parts.

c) An angle bisector divides a(n) _______ of a triangle into two equal parts.

③ Visually identifying and naming pairs of congruent triangles.

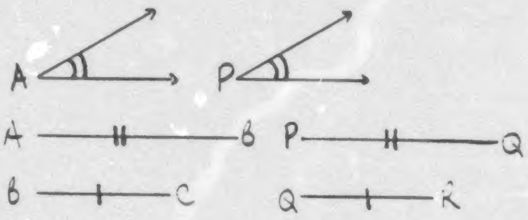
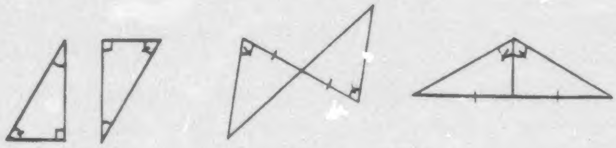
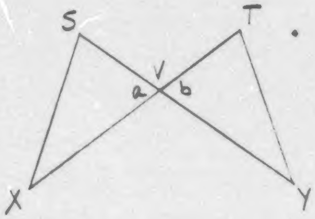
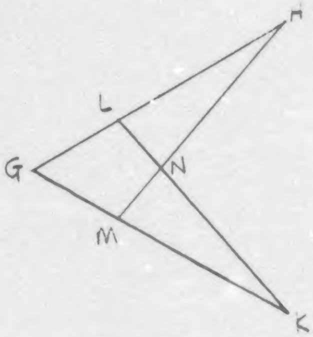
3.



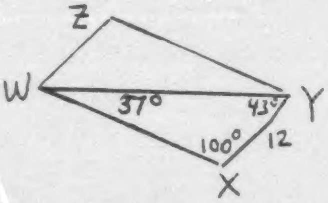
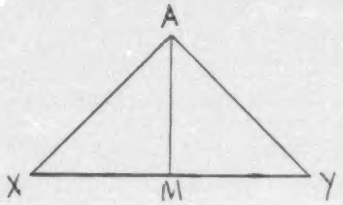
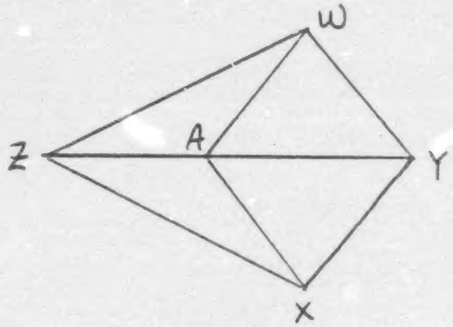
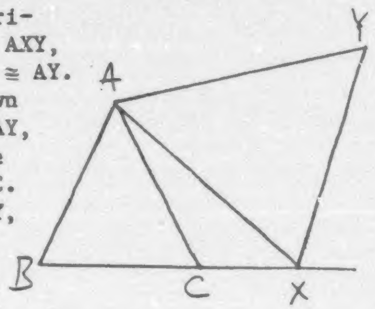


ΔABC ≅ ?   ΔZWX ≅ ?

GEOMETRY C

4. Drawing pairs of triangles, given three of their parts, and observing the cases in which the two triangles must be congruent.	4. Here are some parts of triangles. Draw Δ ABC and Δ PQR. Must the triangles be congruent? 
⑤ Determining whether given information is sufficient to prove two triangles congruent.	5.  For each pair of triangles, decide whether enough information is given to prove the triangles congruent, and if so, devise a proof.
⑥ Proving two given triangles congruent, using SAS, ASA, or SSS.	6.  Given ∠a ≅ ∠b, SV = TV, ∠S ≅ ∠T prove Δ SVX ≅ Δ TVY.
⑦ Proving triangles congruent using SAS, ASA, or SSS along with all other information learned to date.	7.  Given ∠ NMK ≅ ∠NLH and GM GL, prove Δ GHM ≅ Δ GKL.

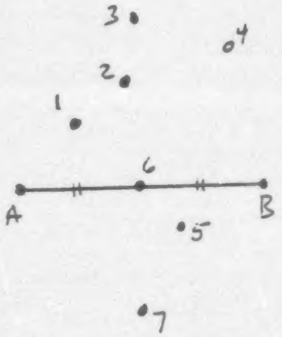
GEOMETRY C

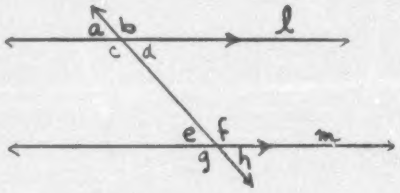
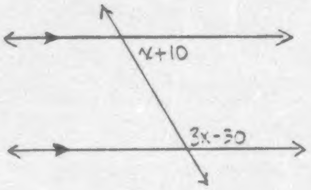
8. Finding measurements by using congruent triangles.	8.  ΔWXY ≅ ΔYZW Give as much information about ΔYZW as you can.
9. Proving segments and angles congruent through the use of congruent triangles.	9.  Given ΔAXY is isosceles, and $\overline{AM}$ is a median, prove ∠X≅∠Y.
10. Proving segments and angles congruent, using congruent triangles and all information learned to date.	10.  Given: WY ≅ XY and WZ ≅ XZ. A is any point on the line YZ. Prove: WA ≅ XA.
11. Making and proving assertions using postulates and theorems learned to date.	11. In isosceles triangles ABC and AXY, AB ≅ AC and AX ≅ AY. It is also known that ∠BAC ≅ ∠XAY, and X is on the extension of BC. Draw in line CY, find and prove two triangles congruent, and prove that CY ≅ BX. 

VOCABULARY

ANGLE BISECTOR OF A TRIANGLE
ANGLE OF A TRIANGLE
BASE
BASE ANGLES OF AN ISOSCELES TRIANGLE
CONGRUENCE
EQUILATERAL TRIANGLE
HYPOTENUSE
IDENTITY CONGRUENCE
INCLUDED ANGLE
INCLUDED SIDE
ISOSCELES TRIANGLE
LEG OF A RIGHT TRIANGLE
MEDIAN OF A TRIANGLE
ONE-TO-ONE CORRESPONDENCE
PERIMETER
RIGHT TRIANGLE
SCALENE TRIANGLE
SIDES OF A TRIANGLE
TRIANGLE
VERTEX
VERTEX ANGLES OF AN ISOSCELES TRIANGLE
VERTEXES OF A TRIANGLE

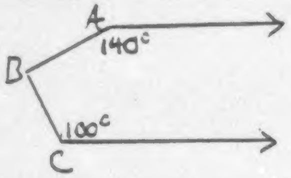
D. PERPENDICULAR AND PARALLEL LINES IN THE PLANE

① Showing visually that points in a plane that are equidistant from the endpoints of a given segment are on the perpendicular bisector of that segment, and showing the converse.	1. Which points appear to be equidistant from the points A and B? 
2. Applying the locus concept to perpendicular bisectors.	2. What is the locus of points in a plane equidistant from two points A and B both in that plane?
③ Using the relationship between points in a plane that are equidistant from the endpoints of a given segment, and the perpendicular bisector of that segment.	3. Point X is on the perpendicular bisector of $\overline{AC}$, and X is not the midpoint of $\overline{AC}$. What kind of triangle is Δ XAC?
4. Proving statements involving perpendicular bisectors.	4. Prove that the perpendicular bisectors of the sides of any triangle are concurrent.
5. Applying the concept of locus to angles.	5. What is the locus of points in a plane equidistant from the sides of an angle in that plane?

6. Knowing as a matter of historical and cultural (as well as mathematical) interest Euclid's fifth postulate (the parallel postulate).	6. State Euclid's fifth or parallel postulate.
⑦ Knowing the relationship among alternate interior angles, corresponding angles, and co-interior angles.	7.  a) Given that line l || line m, name several pairs of congruent angles and several pairs of supplementary angles. b) Name several angle relationships that could be used to prove that l || m.
⑧ Finding angle measures in situations involving parallel lines and transversals.	8. Find the measure of each angle in the picture. 
⑨ Proving assertions using information about parallel lines.	9. Given isosceles triangle ABC with base $\overline{BC}$, and line $\overline{EF} \parallel \overline{BC}$, intersecting $\overline{AB}$ at E and $\overline{AC}$ at F, prove that ΔAEF is isosceles.
⑩ Proving assertions using information about parallel lines along with all other information learned to date.	10. In ΔRST, the bisector of $\angle R$ intersects $\overline{ST}$ at X. The perpendicular bisector of $\overline{RX}$ intersects RT at Y. Prove that $\overline{XY} \parallel \overline{RS}$.
11. Showing the relationships between angles whose sides are mutually parallel or mutually perpendicular.	11. Draw diagrams to answer these questions. a) The sides of $\angle A$ are parallel to the sides of $\angle B$. How are the two angles related? b) The sides of $\angle A$ are perpendicular to the sides of $\angle C$. How are the two angles related?

12. Using the relationships that exist between several parallel lines and several transversals. (See also G-1.)

12. Find the measure of ∠ ABC. Hint: draw a line through B parallel to CD.



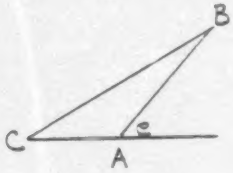
VOCABULARY

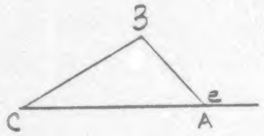
ALTERNATE INTERIOR ANGLES
COINTERIOR ANGLES (INTERIOR ANGLES ON THE SAME SIDE OF THE TRANSVERSAL)
CORRESPONDING ANGLES
DISTANCE BETWEEN A POINT AND A LINE
DISTANCE BETWEEN PARALLEL LINES
LOCUS
PARALLEL LINES
SKEW LINES
TRANSVERSAL

E. TRIANGLES REVISITED

(1.) Demonstrating the relative sizes of an exterior angle and the two remote interior angles of any given triangle. (See also E-4.)

1.





In each triangle, compare
(a) ∠ e and ∠ B
(b) ∠ e and ∠ C

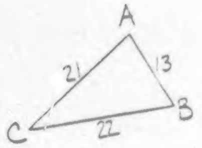
(2.) Using the restrictions imposed on the sides of any triangle by the triangle inequality.

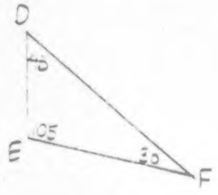
2. You have 6 sets of sticks from which you are asked to build triangles. The lengths are given below. From which sets will you be able to build the triangles? (You are not allowed to break any sticks.)

a) 6, 7, 8
b) 6, 7, 13
c) 10, 3, 6
d) 9, 1, 9
e) 7, 11, 3
f) 5, 5, 1

(3.) Comparing relative sizes of sides and of angles within triangles.

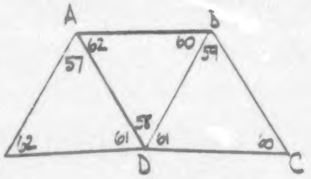
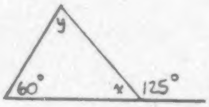
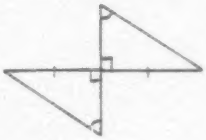
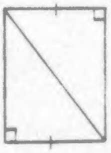
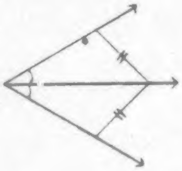
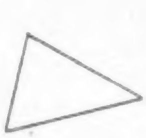
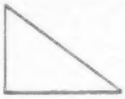
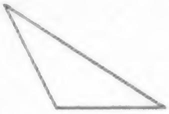
3.



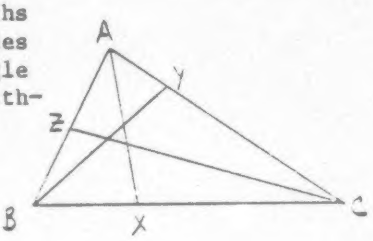
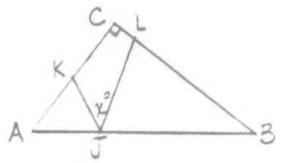


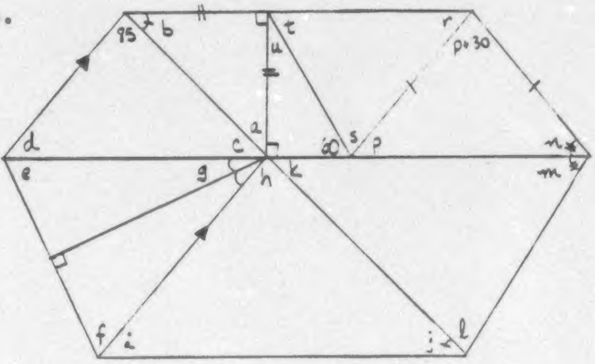
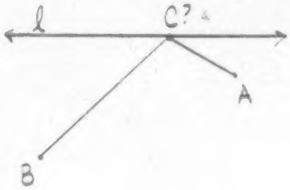
In ∆ ABC, list the three angles in increasing order.
In ∆ DEF, list the three sides in increasing order.

GEOMETRY E

4. Comparing lengths of sides of triangles.	4.  A b 62 57 60 59 58 61 61 62 60 E D C Identify, if possible, a) the longest and b) the shortest segment in the diagram.
⑤ Finding angles of triangles, knowing that the sum of the measures is 180 and the measure of an exterior angle is the sum of the measures of the remote interior angles.	5.  (a) Find the measures of ∠ x and ∠ y. (b) What is true about the two acute angles in any right triangle? (c) Two angles of a triangle have measures 45+a and 60-a. What is the measure of the third angle?
⑥ Using SAA and the hypotenuse-leg theorem in deciding whether given triangles are congruent.	6.    Decide in each case whether enough information is given to prove the two triangles congruent, and if so devise a proof.
⑦ Drawing the altitudes of any given triangle.	7.    Draw the three altitudes of each of these triangles.
⑧ Using the fact that the shortest segment from a point to a line is the perpendicular segment.	8. In any triangle, why is the median drawn from a given vertex certain to be as long as, or longer than, the altitude drawn from the same vertex?

GEOMETRY E

9. Knowing the four concurrence theorems for triangles, knowing the names of the points of concurrency, and knowing something about those points, including which ones must be in the interior of the triangle.	9. The medians of any triangle meet at a point of the way from each vertex to the midpoint of the opposite side. This point is called the ___________. Answer similar questions for the angle bisectors, the perpendicular bisectors of the sides, and the altitudes.
10. Calculating values using the midline (or midsegment) theorem.	10. Δ XYZ has midpoints of sides A, B, and C. If XY = 4, YZ = 5, and ZX = 7, what is the perimeter of Δ ABC?
11. Finding the missing dimensions of right triangles, using the median to the hypotenuse.	11. Find the lengths of as many sides of this triangle as possible without using the Pythagorean theorem. 
12. Applying the locus concept to perpendicular bisectors.	12. What is the locus of points in space that are equidistant from the two points A and B? S. [Superobjective: beginning to draw on one's total geometric background -- not just this week's work -- to solve problems like the next four.]
13. Calculating angles of triangles.	13.  Δ ABC is a right triangle. J is a point on the hypotenuse and AK = AJ and BL = BJ. Prove that x = 45.

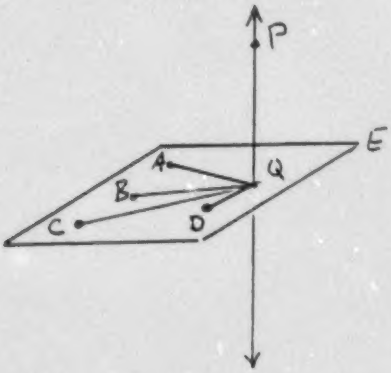
⑭ Calculating angle measures.	14.  95 b t u r p+30 d c a 60 s p n e g h k m f i j l In the figure, points that look collinear are collinear. Segments that look parallel are not parallel unless marked so (—>); similarly for segments and angles that appear to be congruent. Find the measure of every lettered angle. Proceeding in alphabetical order is not necessarily the best approach.
15. Describing the cases in which angles, bisectors, medians, and altitudes of a triangle may be coincident (e.g., when a median may also be an altitude).	15. A triangle may have 9 distinct segments for its 3 angle bisectors, 3 medians, and 3 altitudes. a) How many distinct segments are there when the triangle is isosceles? Equilateral? Right? b) How many distinct segments are possible?
16. Solving problems involving all information learned to date.	16.  a) In triangle ABC, ∠ A-64°. The altitudes meet at H. Find angle BHC. b) In triangle ABC, ∠ A-64°. The angle bisectors meet at I. Find angle BIC.

VOCABULARY

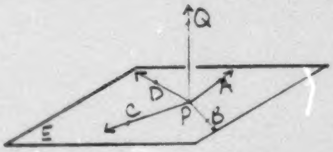
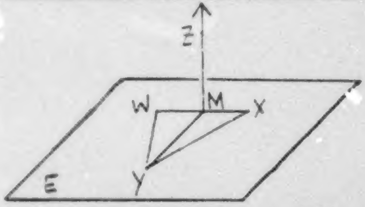
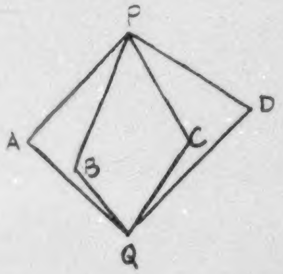
ALTITUDE OF A TRIANGLE
CENTROID
CIRCUMCENTER
CONCURRENCE THEOREMS
EXTERIOR ANGLE OF A POLYGON

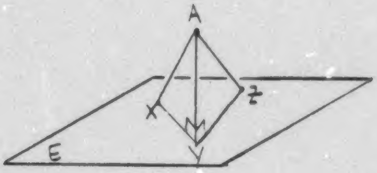
INCENTER
INSCRIBED CIRCLE
MIDLINE OR MIDSEGMENT OF A TRIANGLE
ORTHOCENTER
REMOTE INTERIOR ANGLES

F. PERPENDICULAR AND PARALLEL LINES AND PLANES

① Verifying basic information about parallel and perpendicular lines and planes in the real space of the classroom.	1. Use the walls, edges, and other parts of the classroom to verify the following facts. a) If a plane intersects two parallel planes, then it intersects them in two parallel lines. b) If a line is ⊥ to one of two ‖ planes, then it is ⊥ to the other. c) Two planes ⊥ to the same line are ‖. d) Two lines ⊥ to the same plane are ‖. e) Parallel planes are everywhere equidistant. f) All plane angles of the same dihedral angle are congruent. g) If a line is ⊥ to a plane, then every plane containing the line is ⊥ to the given plane.
② Identifying parallel and perpendicular lines and planes in real space.	2. A classroom includes parts of 6 planes and 12 lines. Give examples of a) parallel lines b) lines perpendicular to planes c) perpendicular planes d) parallel planes e) lines parallel to planes f) skew lines Can there be skew planes?
3. Reading diagrams of three-dimensional situations, using concepts in this unit of material.	3.  $\overline{PQ} \perp$ plane E at Q. Name the angles that must be right angles.

GEOMETRY F

4. Drawing three-dimensional situations.	4. a) Draw a horizontal plane. Draw a line perpendicular to it. Draw another line parallel to the plane and not intersecting the first line. b) Draw two parallel planes. c) Draw two perpendicular planes.
5. Knowing various conditions sufficient to prove a line perpendicular to a plane.	5.  In the figure, what given information would let us prove that $\overline{QP} \perp$ plane E?
6. Proving a line perpendicular to a plane, using current and background information.	6.  Given M is the midpoint of $\overline{WX}$, YW = YX, and $\overline{YM} \perp \overline{MZ}$, where Z is not in plane E (plane E = plane WXY), is $\overline{MZ} \perp$ plane E? If yes, prove it.
7. Solving problems involving lines perpendicular to a plane.	7. A radio antenna 50 feet high is to be anchored with 3 guy wires stretching from the top of the antenna to 3 anchors on the ground equally spaced around the pole. How should the anchor points be placed and the wire measured to ensure that the antenna is perpendicular to the ground?
8. Finding and identifying the perpendicular bisecting plane of a segment.	8.  In this 3-dimensional drawing, PA = QA, PB = QB, PC = QC, and PD = QD. Name four coplanar points.

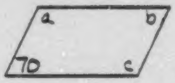
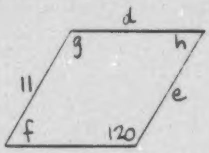
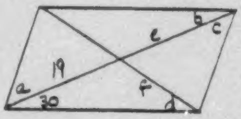
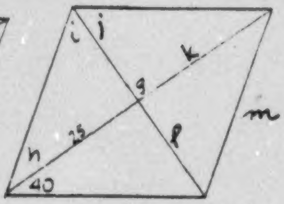
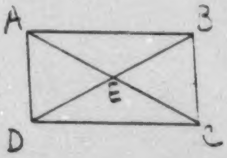
9. Using the fact that the shortest segment from a point to a plane is the perpendicular segment.	9.  Points X, Y, and Z are in plane E. Which of the segments $\overline{AX}$, $\overline{AY}$, and $\overline{AZ}$ is the shortest?
10. Identifying a plane angle of a dihedral angle.	10. Consider your half-opened book as a dihedral angle. Show a plane angle of this dihedral angle.
11. Proving assertions involving parallel planes.	11. a) Prove that if two parallel lines intersect two parallel planes, the segments intercepted by the two planes are congruent. b) Prove that if two parallel planes are intersected by a third plane, the alternate interior dihedral angles are congruent.

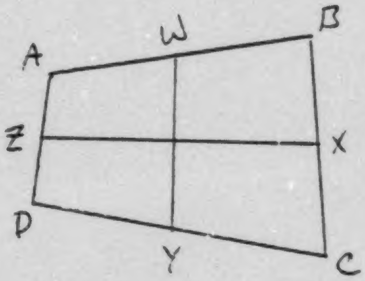
VOCABULARY

DIHEDRAL ANGLE
DISTANCE BETWEEN PARALLEL PLANES
DISTANCE FROM A POINT TO A PLANE
LINE PARALLEL TO A PLANE
LINE PERPENDICULAR TO A PLANE
MEASURE OF A DIHEDRAL ANGLE
PARALLEL PLANES
PERPENDICULAR PLANES
PLANE ANGLE OF A DIHEDRAL ANGLE
PROJECTION OF A LINE INTO A PLANE
PROJECTION OF A POINT INTO A PLANE

G. QUADRILATERALS, REGULAR POLYGONS 1. Sorting (classifying) quadrilaterals: trapezoids, parallelograms, rectangles, rhombi, and squares.	1. a) Draw a rectangle tht is not a square. b) Draw a parallelogram that is not a rhombus. ← c) Draw a trapezoid that is not an isosceles trapezoid. ← d) Fill in the blanks with must, might, or cannot. A parallelogram _______ be a rhombus. A rectangle _______ be a parallelogram. A trapezoid _______ be a square.

GEOMETRY G

② Finding missing measurements in quadrilaterals.	2.  parallelogram  rhombus a) Evaluate a through h. b) In parallelogram ABCD, m∠A = 3x + 3, and m∠B = 10x − 5. Find the numerical measure of ∠D.
3. Using properties of quadrilaterals in practical situations.	3. In laying out the boundaries of a 20 by 40-foot house, you have staked out a parallelogram with sides measuring 20 and 40 feet. How can you turn this parallelogram into a shape that is reliably rectangular?
④ Proving assertions about quadrilaterals.	4. a) Prove that the diagonals of a parallelogram bisect each other. b) Prove that the midpoints of the sides of any quadrilateral are the vertexes of a parallelogram. c) In parallelogram ABCD, AB = 3x + 2, BC = 5x − 20, and CD = 2x + 13. Prove that ABCD is a rhombus.
5. Applying the locus concept to quadrilaterals.	5. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are noncollinear rays. Rhombi are drawn, having one vertex at A, one vertex on $\overrightarrow{AB}$, and one vertex on $\overrightarrow{AC}$. What is the locus of the fourth vertex?
⑥ Finding measurements involving the diagonals of quadrilaterals.	6.   a) Evaluate as many of a through l as possible. b) In rectangle ABCD, AE = 5x − 4 and EC = 3x + 14. Find AC. BD = x^2 + 1. What may we observe? 

7. Solving problems using all information learned to date.	7. a) Can the three angle bisectors of a triangle bisect each other? b) Prove that the 4 angle bisectors in a rectangle form a square. c) ABCD is a quadrilateral with W, X, Y, and Z the midpoints of AB, BC, CD, and DA respectively. Prove that WY ands XZ bisect each other. 
⑧ Demonstrating, visually, basic information about angles of polygons.	8. Draw a pentagon. Draw a picture to show that the sum of the measures of the interior angles is 3 x 180, or 540.
⑨ Solving problems involving angles of regular polygons.	9. In regular nonagon ABCDEFGHI, sides $\overline{AB}$ and $\overline{ED}$ are extended to meet at point P. What is the measure of ∠P? What kind of triangle is △BPD?
⑩ Proving assertions about angles of polygons.	10. Prove that the sum of the angle measures of the exterior angles of a polygon is 360°.

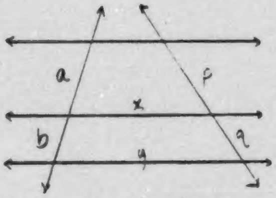
VOCABULARY

ALTITUDE OF A TRAPEZOID
APOTHEM
CONVEX POLYGON
CONVEX QUADRILATERAL
DECAGON
DIAGONAL
DODECAGON
HEPTAGON
HEXAGON
MEDIAN OF A TRAPEZOID
N-GON
NONAGON
OCTAGON
PARALLELOGRAM
PENTAGON
POLYGON
QUADRILATERAL
RECTANGLE
REGULAR
RHOMBUS
SQUARE
TRAPEZOID

H. SIMILARITY

① Expressing the relationships that exist among several parallel lines and more than one transversal, using proportions.

1. In the diagram, the lines that look parallel are parallel. Complete these proportions:



a) $\frac{a}{b} = ?$

b) $\frac{a}{a + b} = ?$

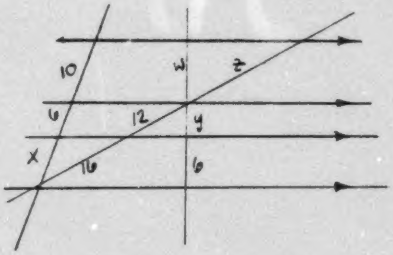
c) $\frac{a}{p} = ?$

d) $\frac{a + b}{p + q} = ?$

② Using the relationships in the preceding item to calculate measurements.

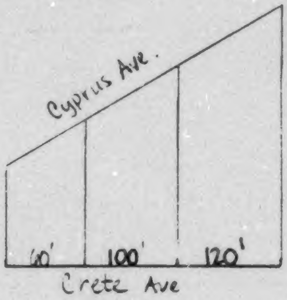
2. Find

w
x
y
z



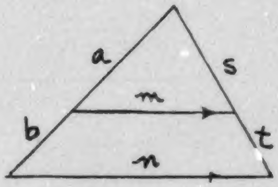
3. Applying the relationships in the preceding item to practical situations.

3. Three business lots cut through a block from Crete Avenue to Cyprus Avenue. The lots' frontages on Crete are shown. If the total frontage of the three lots on Cyprus is 320 feet, give each lot's frontage on Cyprus.



④ Converting measurements on a triangle into proportions.

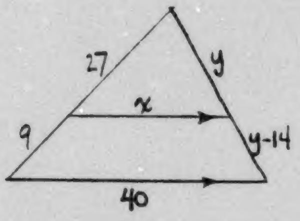
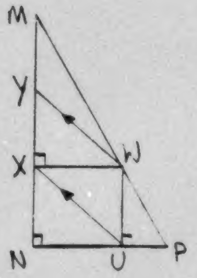
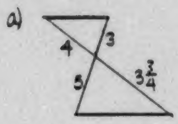
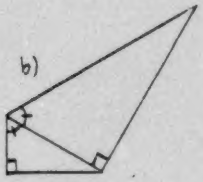
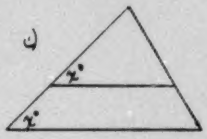
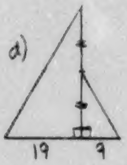
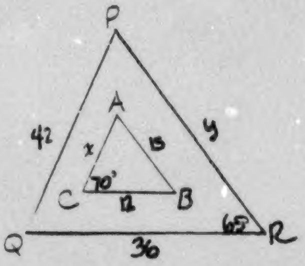
4. Complete the following proportions:



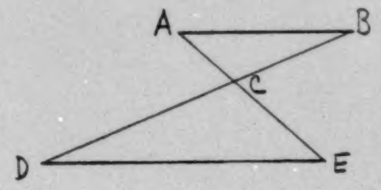
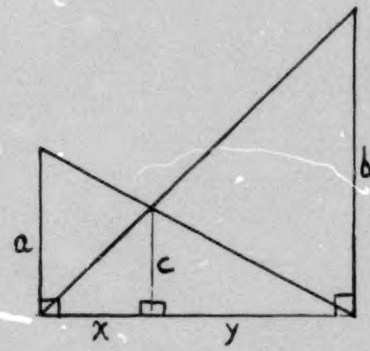
a) $\frac{a}{b} = ?$

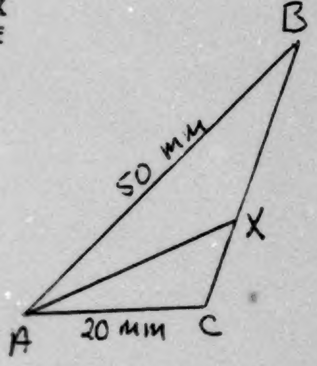
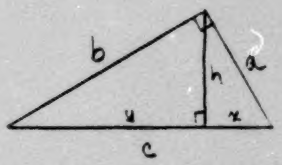
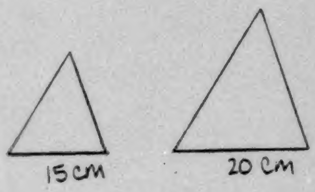
b) $\frac{m}{n} = \frac{a}{?}$

c) $\frac{m}{s} = ?$

⑤ Finding missing measurements on a triangle through the use of proportions.	5. Find the lengths of the remaining 3 segments. 
⑥ Finding similar triangles in a diagram using the SAS or AAA property.	6. Given the figure as marked $(\overline{YW} \parallel \overline{XU})$ List all sets of similar triangles. 
⑦ Determining whether two given triangles are similar, and if so, why.	7.     Decide whether the triangles in each diagram can be proved similar, and if they can be, devise a proof.
⑧ Finding missing measurements in pairs of similar triangles.	8. $\Delta ABC \sim \Delta PRQ$. Find all missing angle measures and lengths of sides.  P 42 y A x 15 70° C 12 B 65° Q 36 R

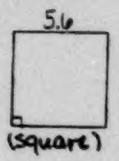
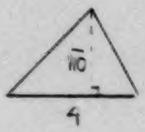
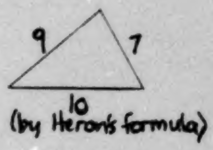
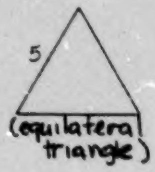
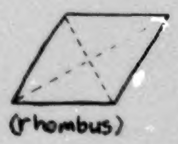
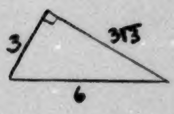
GEOMETRY H

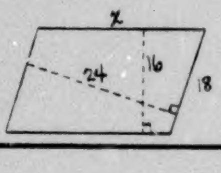
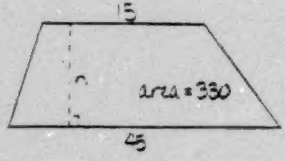
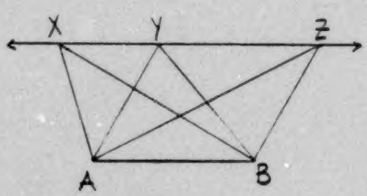
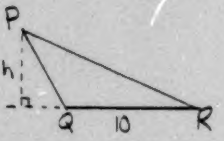
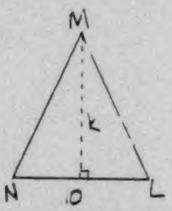
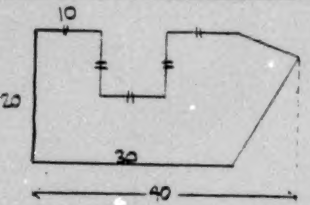
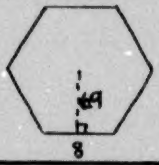
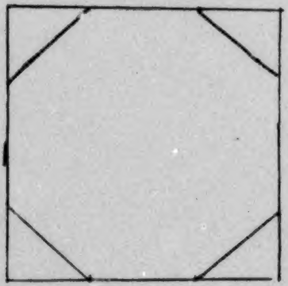
⑨ Proving that two given triangles are similar.	9. Given $\overline{AB} \parallel \overline{DE}$, prove $\Delta ABC \sim \Delta EDC$. 
10. Proving assertions concerning similar triangles.	10. a) Prove that the triangle whose vertexes are the midpoints of the sides of a given triangle is similar to the given triangle. b) Given segment $\overline{XY}$ with midpoint M, ΔXRY and ΔMSY are equilateral triangles on opposite sides of $\overline{XY}$, and $\overline{RS}$ intersects $\overline{XY}$ at T, prove that $\Delta MTS \sim \Delta YTR$ and thus that $TY = \frac{1}{3} XY$.
11. Applying concepts of similarity in practical situations.	11. Explain how you might find the height of a tall building by using similar triangles.
12. Proving assertions concerning parallel lines and transversals.	12. Prove that $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ 
13. Solving problems graphically.	13. If A can rake the yard in 3 hours, and B can do it in 1 hour, how long will it take them if they work together? (Hint, see the preceding item.)

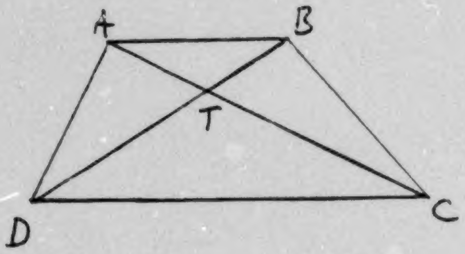
(14) Calculating measurements on a triangle involving angle bisectors.	14. In the figure, $\overline{AX}$ bisects ∠BAC. If BC = 35, find BX and CX.  B 50 mm X A 20 mm C
(15) Finding, in a right triangle, the 3 geometric means in a right triangle created by the altitude to the hypotenuse.	15. In the given right triangle with altitude to the hypotenuse, express each of three segments as geometric means of two other segments. 
(16) Calculating areas of similar figures.	16. The two triangles are similar; the area of the smaller is 135 cm^2. What is the area of the larger? 

VOCABULARY

GEOMETRIC MEAN	PROPORTIONAL SEGMENTS
HARMONIC MEAN	SIMILAR FIGURES

I. AREA	
(1.) Calculating areas of triangles and quadrilaterals using basic formulas and Heron's formula.	1. Find the area of each figure.    9 7 10 (by Heron's formula)  5 (equilateral triangle)  

② Finding missing dimensions of quadrilaterals, given the area.	2. Find x and h.  
③ Finding areas of triangles with common dimensions.	3.  a) Given $\overleftrightarrow{XZ} \parallel \overline{AB}$ and the area of ΔABY is 20. Find the areas of ΔABX and ΔABZ. b) The ratio of h to k is 1:2. The area of ΔLMN is 17. What is the area of ΔPQR?  
④ Finding areas of complex polygonal regions.	4. Find the area of this region. Angles that look like right angles are right angles. 
⑤ Finding areas of regular polygons.	5. Find the area of this regular hexagon. 
6. Proving assertions about area.	6. Prove that the diagonals of a parallelogram form 4 triangles having equal area.
7. Finding areas of regular polygons, using one's mathematics background.	7. a) The medians of a triangle are 15 cm, 15 cm, and 24 cm. Find the area of the triangle. b) The four corners of a square are cut off to form a regular octagon. If the sides of the square are 10 cm, find the area of the octagon. 

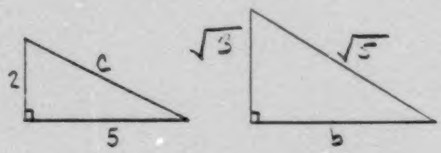
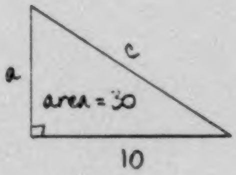
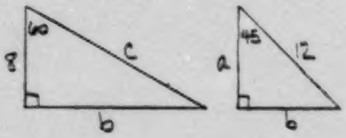
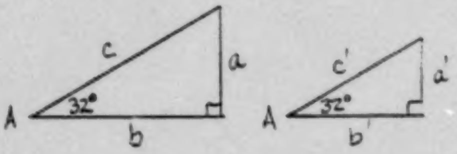
8. Solving problems about area, using all one's mathematics background.	8. a) ABCD is a trapezoid in which AB = 12 and DC = 20. If the area of ΔABT is 9 cm^2, find the area of ΔDTC, ΔATD, and ΔBTC.  b) Given any two parallelograms lying anywhere in the same plane, show how to draw one line that will separate each parallelogram into two regions whose areas are equal.

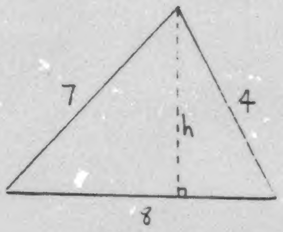
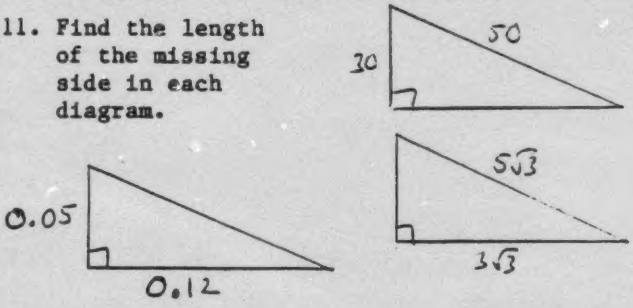
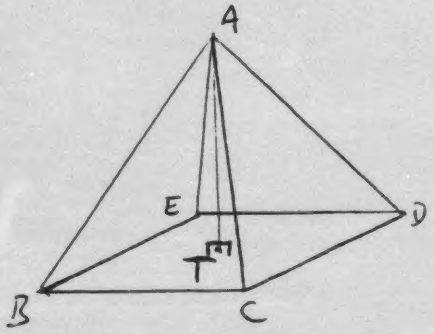
VOCABULARY

AREA
POLYGONAL REGION
TRIANGULAR REGION

J. THE PYTHAGOREAN THEOREM

① Stating the Pythagorean theorem in terms of areas.	1. Draw a right triangle with legs a and b and hypotenuse c. Show what $a^2 + b^2 = c^2$ means in terms of area.
2. Relating the Pythagorean theorem to the coordinate plane.	2. Show how the formula for the distance between points (x_1, y_1) and (x_2, y_2) is related to the Pythagorean theorem.
3. Stating a proof of the Pythagorean theorem.	3. Give a proof, visual or numerical, of the Pythagorean theorem.

④ Finding the missing sides of right triangles.	5.   Find the missing lengths.
⑤ Determining whether a given triangle is a right triangle.	5. A triangle has sides of lengths 5, 17, and 18. Is it a right triangle?
⑥ Finding missing dimensions of the 30-60 right triangle and the isosceles right triangle.	6. Find the missing lengths. 
7. Using similar right triangles to define the sine, cosine, and tangent of an angle.	7. $\frac{a}{b} = \frac{a'}{?}$  If in a third right triangle the ratio of the two legs a/b is the same, what is the measure of angle A? When we link this number, a/b, ∠A, what do we call it?
8. Proving assertions involving the Pythagorean theorem.	8. a) If a and b are the lengths of the legs of a right triangle, c is the length of the hypotenuse, and k is any positive number, show that a triangle whose sides have lengths ka, kb, and kc is also a right triangle. b) ABCD is a quadrilateral whose diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular. Show that $(AB)^2 + (CD)^2 = (BC)^2 + (DA)^2$.

⑨. Finding an altitude of a given triangle.	9. Find h. 
⑩ Using the Pythagorean theorem in practical situations.	10. A piece of wire for a radio antenna is to be strung from the top of a house to the top of the garage 40 feet away. The garage roof is 15 feet high, and the house is 45 feet high. The wire must be at least how long?
11. Knowing several right triangles whose sides have whole numbers as lengths.	11. Find the length of the missing side in each diagram. 
12. Finding dimensions of solids.	12. a) A cube has an edge of 8 inches. Find the length of a diagonal of a face and of a diagonal of the cube. b) In the figure below, ABCDE is a pyramid with a square base. The sides of the base are all 12 cm and the length of the altitude AT is 10 cm. Find the length AC. 

⑬ Solving problems involving the Pythagorean theorem.

13. a) The diagonals of a rhombus have lengths of 10 and 24. Find the perimeter of the rhombus.

b) The hypotenuse of an isosceles right triangle is 14. What is the area of the triangle?

c) A base angle of an isosceles triangle has a measure 30. The length of one of the congruent sides is 12. What is the area of the triangle?

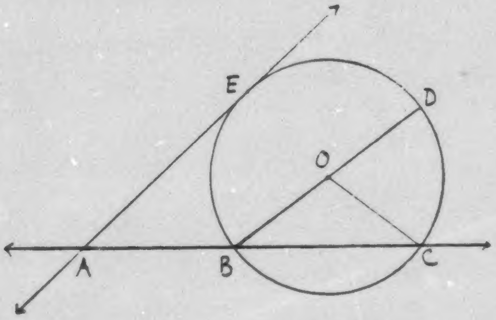
VOCABULARY

COSINE
ISOSCELES RIGHT TRIANGLE
SINE
TANGENT
30-60 RIGHT TRIANGLE

K. CIRCLES, SPHERES

① Finding and naming in a diagram the basic figures associated with circles.

1.



In this picture, find and name

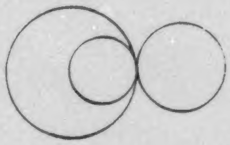
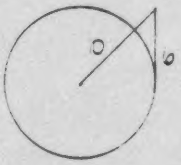
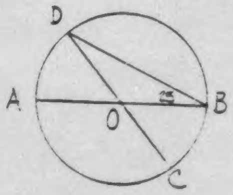
a radius
a diameter
a chord
a secant
a tangent
a major arc
the center of the circle
a semicircle
a minor arc
a central angle
an inscribed angle
the arc intercepted by ∠BOC

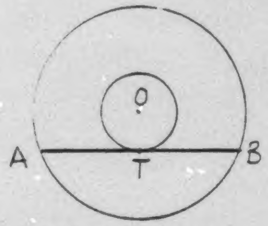
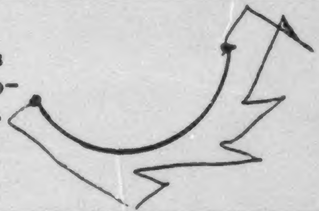
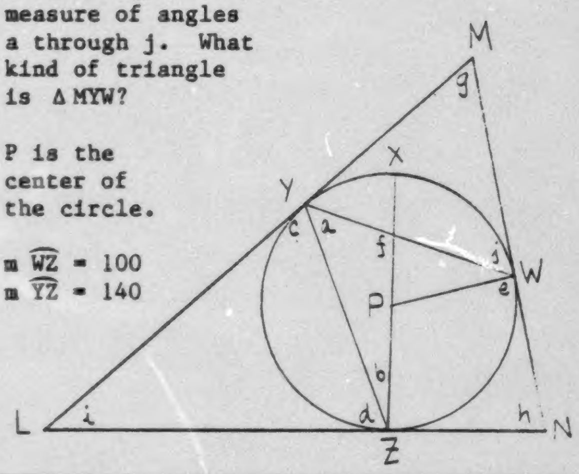
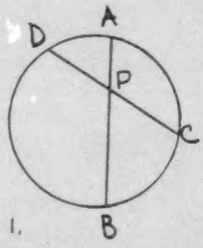
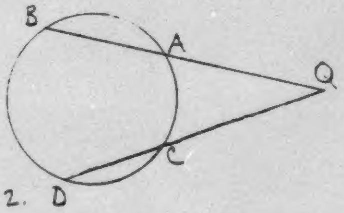
2. Showing the basic figures associated with spheres.

2. Using a globe and a piece of cardboard, indicate

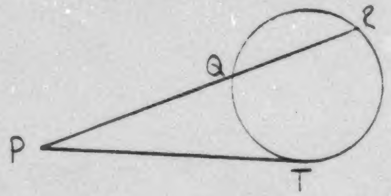
a great circle of a sphere
a hemisphere
a secant plane
a tangent plane
Show that the intersection of a plane and a sphere is a point or a circle.

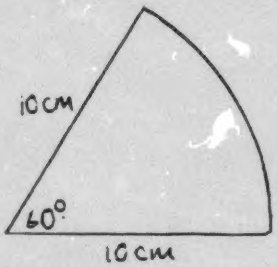
GEOMETRY K

③ Applying the locus concept to circles and spheres.	3. Given point A, what is the locus of points that are 10 cm from A a) in a plane containing A b) in a plane not containing A c) in space [Note: (b) has 3 cases.]
4. Solving purely visual problems involving circles.	4. Here are three circles of different radii, with each circle tangent to the other two. How many other arrangements are possible so that each circle is tangent to both of the others? 
⑤ Performing calculations involving circles, chords, and tangents.	5. a) a six-inch chord is 4 inches from the center of the circle. What is the diameter of the circle? b) A point is 10 inches from the center of a circle. A segment that is tangent to the circle and has the point at one end is 6 inches long. What is the radius of the circle? 
⑥ Performing calculations involving arcs.	6. Diameters $\overline{AB}$ and $\overline{CD}$ intersect at O. If $m \angle ABD = 25$, find the measure of all minor arcs in the circle. 

7. Proving assertions involving circles, chords, tangents, secants, and arcs.	7. a) Prove that no three points on a circle are collinear. b) Given two concentric circles with center C, and chord $\overline{AB}$ tangent to the smaller circle at T, prove that T is the midpoint of $\overline{AB}$. 
8. Applying one's knowledge of circles to practical situations.	8. A piece of the rim of a broken vase is found in an archaeological expedition. Find the diameter of the rim. 
⑨ Calculating the measure of angles, both of whose sides intersect a given circle.	9. In the figure, find the measure of angles a through j. What kind of triangle is ΔMYW? P is the center of the circle. $m\ \widehat{WZ} = 100$ $m\ \widehat{YZ} = 140$  M g X Y c a f j W e P b L i d h N Z
10. Performing calculations involving the power of a point with respect to a circle.	10.  1.  2. a) If PA=3 and PB=8 and PC=6, then PD=? b) If QA=5 and AB=5 and QC=6, then CD=? c) What is the power of Q with respect to the circle shown?

GEOMETRY K

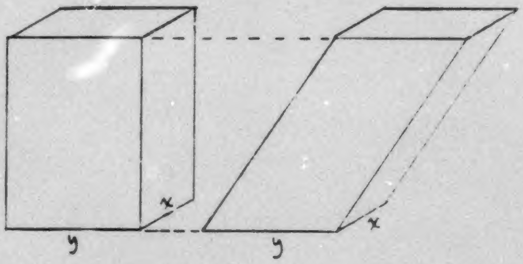
11. Proving assertions involving the power of a point with respect to a given circle.	11.  $\overline{PT}$ is a tangent segment. Prove that (PQ)(PR) = $(PT)^2$. (Hint: use similar triangles.)
12. Solving problems involving circles, on the coordinate plane.	12. Find the center and radius of the circle whose equation is a) $x^2 + y^2 = 49$ b) $x^2 + 6x + y^2 = 0$
13. Performing calculations involving inscribed (cyclic) polygons.	13. Quadrilateral ABCD is inscribed in a circle. $m\angle A = 100$, and $m\angle B = 65$. Give the measures of the other two angles.
14. Proving assertions involving inscribed and circumscribed polygons.	14. a) The quadrilateral MNPQ circumscribes a circle. Prove that MN + PQ = NP + QM. b) Prove that if two opposite sides of an inscribed quadrilateral are congruent, then the other two sides are parallel.
⑮ Performing calculations involving the circumference of a circle.	15. a) The radius of a circle is 6. What is the circumference? b) The circumference of a circle is [illegible]0. What is the diameter?
⑯ Performing calculations involving the area of a circle.	16. a) The radius of a circle is 25. What is the area of the circle? b) A circle is inscribed in a square. The radius of the circle is 6. What is the area of the region outside the circle but inside the square?

17. Performing calculations involving the length of an arc of a circle.	17. A circle of radius 12 has an arc of measure 20°. Find the length of the arc.
18. Performing calculations involving the area of a sector of a circle.	18. Find the area of the sector below. 
19. Proving assertions, using all material learned to date.	19. a) Prove that if in a circle two chords bisect each other, they are both diameters. b) Prove that if $\overline{PR}$ and $\overline{ST}$ are any two diameters of a sphere, then the quadrilateral PSRT is a rectangle.
20. Solving problems using all information learned to date.	20. According to the Guinness Book of World Records, the tops of the towers of the Verrazano Narrows Bridge are $1\frac{3}{8}$ inches farther apart than are the bases. What additional information would be needed to calculate the radius of the earth, and how would you perform such calculations?

VOCABULARY

ARC
ARC INTERCEPTED BY AN ANGLE
ARC LENGTH
AREA OF A CIRCLE
CENTER
CENTER OF AN ARC
CENTRAL ANGLE
CHORD
CIRCLE
CIRCULAR REGION
CIRCUMFERENCE
CIRCUMSCRIBED POLYGON
COMMON EXTERNAL TANGENT
COMMON INTERNAL TANGENT
CONCENTRIC
CONCYCLIC POINTS
CONGRUENT ARCS
CYCLIC QUADRILATERAL
DEGREE MEASURE OF AN ARC
DIAMETER
GREAT CIRCLE
INSCRIBED ANGLE
INSCRIBED POLYGON
MAJOR ARC
MINOR ARC
PI
POINT OF TANGENCY
POWER OF A POINT
RADIUS
SECANT LINE
SECANT SEGMENT
SECTOR
SEMICIRCLE
SPHERE
TANGENT LINE
TANGENT PLANE TO SPHERE
TANGENT SEGMENT

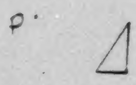
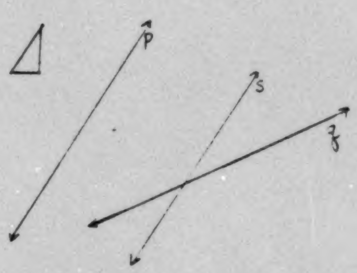
L. SOLIDS	
① Calculating the surface area and volume of a given prism.	1. a) Find the total surface area and volume of a cube whose edge is 2 miles. b) The bases of a prism are isosceles trapezoids, and the height of the prism is 10 cm. The bases of the trapezoids are 5 cm and 13 cm, and the height is 3 cm. Find the total surface area and volume of the prism.
② Calculating the surface area and volume of a given pyramid.	2. A pyramid's base is a square 10 measuring 10 inches on each side. The height of the pyramid is 12 inches. Find the lateral area, the total surface area, and the volume of the pyramid.
3. Calculating the surface area and volume of a frustum of a pyramid.	3. The base edges of a frustum of a regular square pyramid are 4 inches and 10 inches, and the slant height is 5 inches. Find the total surface area and the volume of the frustum.
④ Calculating the surface area and volume of a given cylinder.	4. A cylinder has a radius of 3 inches and a height of 4 inches. Give its surface area (in terms of pi) and the volume (also in terms of pi).
⑤ Calculating the surface area and volume of a given cone.	5. A cone has a slant height of 12 and a radius of 5. Find its lateral area, its total area, and its volume.
6. Calculating the surface area and volume of a given frustum of a cone.	6. The slant height of a frustum of a cone is 5 inches and the radii of its bases are 3 and 4 inches. Find the lateral area and the volume.
⑦ Calculating the surface area and volume of a given sphere.	7. A classroom globe has a radius of 20 cm. Find its surface area and volume.

8. Calculating the surface area and volume of a given inscribed solid or circumscribed solid.	8. A cone is inscribed in a sphere with a radius of 7 inches. The height of the cone is 9 inches. Find the volume of the cone.
9. Using Cavalieri's Principle to compute volumes.	9.  The volume of the solid on the left is 640 cm^3. What is the volume of the solid on the right? How does Cavalieri's Principle help in the solution?
10. Calculating the surface areas and volumes of similar solids.	10. The ratio of corresponding edges of two similar rectangular prisms is 3:4. The surface area of the smaller is 94 in^2 and its volume is 60 in^3. What are the surface area and volume of the larger prism?
11. Solving problems involving surface area and volume.	11. a) A cone is made from a sector of a circle whose radius is 10 inches. The central angle of the sector is 108°. Find the radius of the cone. b) A sphere is packed in a cubical box, into which it fits exactly. What percent of the volume of the box is occupied by the sphere?
12. Solving practical problems involving surface area and volume.	12. A cylindrical piece of pipe is 4 mm thick. Its outside radius is 8 cm. Its length is 30 cm. Find the volume of metal in the pipe.
13. Solving problems involving volume and one's mathematical background.	13. The total surface area (including ends) of a cylindrical can is 192 pi in^2, and its height is 10 in. Find the radius of the can.

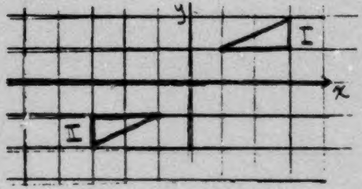
VOCABULARY

ALTITUDE OF A PRISM OR CYLINDER
BASE OF A CONE OR PYRAMID
BASE OF A PRISM OR CYLINDER
CONE
CROSS SECTION OF A PRISM, CYLINDER, CONE, PYRAMID
CUBE
CYLINDER
EDGES OF A SOLID
FRUSTUM OF A CONE OR PYRAMID
GEODESIC
LATERAL EDGE
LATERAL FACE
LATERAL SURFACE
PARALLELEPIPED
PRISM
PYRAMID
RECTANGULAR PARALLELEPIPED (CUBOID)
RIGHT PRISM
SLANT HEIGHT OF A CONE OR PYRAMID
VERTEX OF A CONE OR PYRAMID
VOLUME

M. TRANSFORMATIONS

1. Producing the image of a given figure under a given reflection, rotation, or translation either without using coordinates or in the coordinate plane.	1. a) Find the image of the triangle under a 45° rotation around P.  b) Find the image of the segment whose endpoints are (2,-4) and (-3,0) under a reflection in the line x = 5.
2. Producing the image of a given figure under a composite transformation.	2. a) Find the image of the segment whose endpoints are (2,-4) and (-3,0) under a reflection in the x-axis followed by a reflection in the line y = 2.  b) Find the image of the triangle with vertices (6,2), (6,-1), and (4,-1) under reflection in the x-axis followed by a rotation of 90° about the origin.

GEOMETRY M, N

3. Given the pre-image and image of a figure, producing a succession of isometries carrying the pre-image to the post-image.	3. ΔI has been carried to ΔII by certain reflections. Give the specific equations of reflecting lines and the order of reflections. 
4. Knowing the properties of a figure that are invariant under a reflection, rotation, or translation.	4. Which of these properties of a figure are left invariant by a rotation? a) orientation b) collinearity of points c) angle measure d) parallelism e) distance f) area g) volume

VOCABULARY

AXIS OF SYMMETRY
COMPOSITES OF TRANSFORMATIONS
IMAGE
INVERSE TRANSFORMATION
LINE OF SYMMETRY
ORIENTATION
PRE-IMAGE
REFLECTING LINE
REFLECTION
ROTATION
TRANSFORMATION
TRANSLATION

N. LOGIC

1. Distinguishing between direct and indirect arguments.	1. A "guaranteed solid gold" ring is turning your finger green. You are on your way to the jeweler to prove to him that the ring is not gold. What argument will you use? Is it direct or indirect?
2. Giving the converse, inverse, and contrapositive of a given statement, and noting which pairs are logically equivalent.	2. "If you are rich, then you have lots of friends." Give the converse, inverse, and contrapositive of this statement. Put the four statements into two pairs of logically equivalent statements.

3. Rewriting statements using standard logical connectives.	3. Rewrite the following statement a) using "if" and "then" and b) using only "if." The early bird gets the worm.

VOCABULARY

CONCLUSION
CONTRAPOSITIVE
CONVERSE
COROLLARY
COUNTEREXAMPLE
DEDUCTION
DEFINITION
DIRECT PROOF
HYPOTHESIS
IF AND ONLY IF (IFF)
IF, ONLY IF
IMPLIES
INDIRECT PROOF
INVERSE
POSTULATE
PROOF
QUOD ERAT DEMONSTRANDUM
THEOREM
UNDEFINED TERMS

O. GEOMETRIC CONSTRUCTIONS

1. a segment congruent to a given segment
2. a regular hexagon given one side
3. the perpendicular bisector of a given line segment (= midpoint)
4. the perpendicular bisectors of the sides of a given triangle
5. the circle circumscribed around a given triangle (circumcenter)
6. the circle passing through 3 given noncollinear points
7. the medians of a given triangle (centroid)
8. an angle congruent to a given angle
9. the bisector of a given angle
10. the angle bisectors of a given triangle (incenter)
11. a triangle congruent to a given angle
12. a 60° angle
13. a 30° angle
14. the perpendicular to a line through a given point on the line
15. the perpendicular to a line through a given external point
16. the altitudes of a triangle (orthocenter)
17. the circle inscribed in a given triangle
18. a 90° angle
19. a 45° angle
20. the bisectors of the exterior angles of a given triangle
21. the excenters of a given triangle
22. escribed circles
23. the line parallel to a given line through a given point
24. to divide a given segment into n congruent segments
25. a rectangle (or square) given its sides
26. a triangle with area equal to that of a given rectangle
27. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, an so on, given a segment of length 1
28. the fourth proportional to 3 given numbers
29. the arithmetic mean of 2 given numbers
30. the geometric mean of 2 given numbers
31. the harmonic mean of 2 given numbers
32. the three means, compared in one drawing
33. the center of a circle, given an arc
34. the tangent to a given circle at a given point on the circle
35. the tangent to a given circle through a given external point
36. the common external tangents to 2 given circles
37. the common internal tangents to 2 given circles
38. an arc in which a given angle is inscribed
39. a circle orthogonal to a given circle
40. the radical axis of 2 given circles
41. the 9-point circle

P. GEOMETRY AND CAREERS

Investigating one job from each of three occupational clusters in which the skills learned in Geometry can be applied.

GEOMETRY COURSE VOCABULARY

Acute Angle	A
Adjacent Angles	A
Alternate Interior Angles	D
Altitude of a Prism or Cylinder	L
Altitude of a Trapezoid	G
Altitude of a Triangle	E
Angle	A
Angle Bisector	A
Angle Bisector of a Triangle	C
Angle of a Triangle	C
Apothem	G
Arc	K
Arc Intercepted by an Angle	K
Arc Length	K
Area	I
Area of a Circle	K
Axis of Symmetry	M
Base	C
Base Angles of Isosceles Triangle	C
Base of a Cone or Pyramid	L
Base of a Prism or Cylinder	L
Between	A
Bisect	A
Bisector	A
Center	K
Center of an Arc	K
Central Angle	K
Centroid	E
Chord	K
Circle	K
Circular Region	K
Circumcenter	E
Circumference	K
Circumscribed Polygon	K
Cointerior Angles (Interior Angles on the Same Side of the Transversal)	D
Collinear	A
Common External Tangent	K
Common Internal Tangent	K
Complement of an Angle	A
Complementary Angles	A
Composites of Transformations	M
Concentric	K
Conclusion	N
Concurrence Theorems	E
Concurrent	A
Concyclic Points	K
Cone	L
Congruence	C
Congruent Angles	A
Congruent Arcs	K
Congruent Segments	A
Contrapositive	N
Converse	N
Convex Polygon	G
Convex Quadrilateral	G
Convex Set	B
Coordinate	A
Coordinate System	A
Coplanar	A
Corollary	N
Corresponding Angles	D
Cosine	J
Counterexample	N
Cross Section of a Prism, Cylinder, Cone, Pyramid	L
Cube	L
Cyclic Quadrilateral	K
Cylinder	L
Decagon	G
Deduction	N
Definition	N
Degree	A
Degree Measure of an Arc	K
Determine	B
Diagonal	G
Diameter	K
Dihedral Angle	F
Dimension	A
Direct Proof	N
Distance	A
Distance between a Point and a Line	D
Distance between Parallel Lines	D
Distance between Parallel Planes	F
Distance from a Point to a Plane	F
Distinct	A
Dodecagon	G
Edge of a Half-Plane	B
Edges of a Solid	L
Endpoint	A
Equidistant	A
Equilateral Triangle	C
Exterior Angle of a Polygon	E
Exterior of an Angle	A
Face of a Half-Space	B
Frustum of a Cone or Pyramid	L
Geodesic	L
Geometric Mean	H

GEOMETRY COURSE VOCABULARY

Great Circle	K
Half-Plane	B
Half-Space	B
Harmonic Mean	H
Heptagon	G
Hexagon	G
Hypotenuse	C
Hypothesis	N
Identity Congruence	C
If and Only If (IFF)	N
If, Only If	N
Image	M
Implies	N
Incenter	E
Included Angle	C
Included Side	C
Indirect Proof	N
Inscribed Angle	K
Inscribed Circle	E
Inscribed Polygon	K
Interior of an Angle	A
Inverse	N
Inverse Transformation	M
Isosceles Right Triangle	J
Isosceles Triangle	C
Lateral Edge	L
Lateral Face	L
Lateral Surface	L
Leg of a Right Triangle	C
Line	A
Linear Pair of Angles	A
Line of Symmetry	M
Line Parallel to a Plane	F
Line Perpendicular to a Plane	F
Locus	D
Major Arc	K
Measure of a Dihedral Angle	F
Measure of an Angle	A
Median of a Trapezoid	G
Median of a Triangle	C
Midline or Midsegment of a Triangle	E
Midpoint	A
Minor Arc	K
N-Gon	G
Nonagon	G
Obtuse Angle	A
Octagon	G
One-to-One Correspondence	C
Opposite Rays	A
Orientation	M
Orthocenter	E
Parallel Lines	D
Parallel Planes	F
Parallelepiped	L
Parallelogram	G
Pentagon	G
Perimeter	C
Perpendicular Lines	A
Perpendicular Planes	F
Pi	K
Plane Angle of a Dihedral Angle	F
Point of Tangency	K
Polygon	G
Polygonal Region	I
Postulate	N
Power of a Point	K
Preimage	M
Prism	L
Projection of a Line into a Plane	F
Projection of a Point into a Plane	F
Proof	N
Proportional Segments	H
Protractor	A
Pyramid	L
Quadrilateral	G
Quod Erat Demonstrandum	N
Radius	K
Ray	A
Rectangle	H
Rectangular Parallelepiped (Cuboid)	L
Reflecting Line	M
Reflection	M
Reflex Angle	A
Regular	G
Remote Interior Angles	E
Rhombus	G
Right Angle	A
Right Prism	L
Right Triangle	C
Rotation	M
Scalene Triangle	C
Secant Line	K
Secant Segment	K
Sector	K
Segment	A
Semicircle	K
Separate	B
Sides of an Angle	A

GEOMETRY COURSE VOCABULARY

Sides of a Triangle	C
Similar Figures	H
Sine	J
Skew Lines	D
Slant Height of a Cone or Pyramid	L
Space	A
Sphere	K
Square	G
Supplementary Angles	A
Supplement of an Angle	A
Tangent	J
Tangent Line	K
Tangent Plane to Sphere	K
Tangent Segment	K
Theorem	N
30-60 Right Triangle	J
Transformation	M
Translation	M
Transversal	D
Trapezoid	G
Triangle	C
Triangular Region	I
Undefined Terms	N
Vertex	C
Vertex Angles of an Isosceles Triangle	C
Vertexes of a Triangle	C
Vertex of a Cone or Pyramid	L
Vertex of an Angle	A
Vertical Angles	A
Volume	L

ALGEBRA II AND TRIGONOMETRY OBJECTIVES

Introduction

These objectives follow Geometry in the high school mathematics sequence, and serve as preparation for Advanced Mathematics courses for students who elect to go further in the mathematics progression. The Algebra II Objectives are divided into the following sections:

A. Real Numbers: Their Properties and Operations
B. Equations and Inequalities Involving One Variable
C. Linear Equations
D. Polynomials and Factoring
E. Relations, Functions, Transformations
F. Fractional Expressions and Equations
G. Exponents, Powers, and Roots
H. Complex Numbers
I. Quadratic Equations and Quadratic Functions
J. Systems of Equations and Systems of Inequalities
K. Polynomial Functions
L. Equations of Second Degree
M. Exponential and Logarithmic Functions
N. Sequences and Series
O. Matrices and Determinants
P. Combinatorial Algebra: Probability
Q. Algebra II and Careers

The Trigonometry Objectives are divided into the following sections:

A. Introduction to Trigonometry
B. Angle Measurement
C. Solutions of Triangles
D. Identities, Equations, and Trigonometric Functions of Complex Numbers
E. Trigonometry and Careers

ALGEBRA II

A. REAL NUMBERS: THEIR PROPERTIES AND OPERATIONS	
① Graphing real numbers on a number line and comparing their values.	1. Graph these real numbers on a number line. Then list the numbers in order from least to greatest. (a) -2, -3.5, -4.5, -1 (b) $-\frac{1}{4}, -\frac{3}{4}, \frac{3}{4}, -\frac{7}{4}$ (c) -2.4, -2.1, -2, -2.15
② Evaluating simple open expressions, given the value of specified variables.	2. Evaluate the following: (a) $x^2 - 3yz$; $x = -2$, $y = 3$, $z = -1$ (b) $-x + y^z$; $x = -2$, $y = 3$, $z = -1$ (c) $\frac{(ac)^d}{(b^2)} + \sqrt{a^2b^2}$; $a = 2$, $b = 3$, $c = 1$, $d = 0$
③ Performing all four operations on real numbers and recognizing undefined divisions.	3. Evaluate the following: (a) $(-9+20) + (-13.7) =$ (b) $6(-7) - (-9)(-5) =$ (c) $-3(-4x^3+2x-1) =$ (d) $\frac{[(-9)(8)-(-6)^2] + 4}{(2-11)(4-5)} =$
④ Using the order of operations to simplify numerical statements.	4. Simplify: (a) $2\cdot4^3 - 3\cdot4^2 + 5\cdot4 - 7$ (b) $\frac{3^2\cdot6 \div 3}{2\cdot1 + (3+1)^2}$
⑤ Simplifying algebraic expressions.	5. Simplify: (a) $14x - [5x-(6y-3)]$ (b) $\frac{-80x^2y^5}{15x^3y^2}$ (c) $(x^3)^2 - x^4(x^2+1) + (x^2)^2$ (d) $\frac{7x^{-6}y^{10}}{-56x^2y^{-2}}$

ALGEBRA II A

⑥ Simplifying expressions involving absolute value.	6. Simplify: (a) $\|-20x^3y^2\|$ (b) $(-18+\|-16\|)-\|24\|$ (c) $\|-7+15\|+\|-8-11\|$
⑦ Identifying the field properties that are illustrated in a given sentence.	7. Identify the field property that is illustrated by each sentence: (a) $5(b+a) = (b+a)5$ (b) $\frac{1}{(x+y)} \cdot (x+y) = 1$ (c) $2r+3r = (2+3)r$
⑧ Applying field properties of real numbers in simplifying expressions for sums, products, differences, and quotients.	8. State the field property that justifies each step in the following: $(3x)(2x) = (3x\cdot2)x$? $= (2\cdot3x)x$? $= [(2\cdot3)x]x$? $= (6x)x$? $= 6(x\cdot x)$? $= 6x^2$?
9. Applying field properties of real numbers to supply reasons for steps in proofs.	9. Give the reasons for the steps in each proof. If $c \neq 0$, $(bc)\frac{1}{c} = b$ 1. $c \neq 0$? 2. $\frac{1}{c}$ is a real number ? 3. $(bc)\frac{1}{c} = b(c\cdot\frac{1}{c})$? 4. $= b\cdot1$? 5. $= b$? 6. $\therefore (bc)\frac{1}{c} = b$?
10. Evaluating problems with the use of calculators.	10. Use a calculator to evaluate these problems: (a) (-14,773)(-27) (b) (-0.56004)(0.003) (c) (-14,773) ÷ (27) (d) $\frac{(52)(-7)(-12)}{17-(-13)}$

B. EQUATIONS AND INEQUALITIES INVOLVING ONE VARIABLE	
① Simplifying polynomial expressions.	1. Simplify by adding and/or subtracting similar terms. (a) $(2x^2-x+3)-(4-3x+x^2)$ (b) $(2t^3+8t^2-9)-(4t^3-8t-9)$ (c) $3(v^2-2)-15(\frac{1}{3}-v^2)$
② Solving first-degree equations in one variable.	2. Solve for x: (a) $3(x-2) = 2x-(6-x)$ (b) $(\frac{1}{2})x+x = x-(\frac{1}{2})(2-x)$
③ Expressing English sentences in mathematical equations and solving them.	3. The lengths in meters of the sides of a triangle are consecutive integers. The shortest side is y meters long. What is the perimeter?
④ Applying first-degree equations in one variable to solve word problems.	4. A piece of wire 10 meters long is cut into two pieces so that one piece is two-fifths as long as the other. Find the length of each.
⑤ Solving linear inequalities in one variable and graphing the solution on a number line.	5. Solve and graph the solution on a number line. (a) $10-x < 6$ (b) $2(v+3)-5 > 5(v-1)$
⑥ Solving compound sentences of inequalities in one variable.	6. Solve and graph the solution on a number line. (a) $-5 \leq 3x-2 < 4$ (b) $7-2m \geq 5$ and $11+2m > 5-m$
⑦ Solving first-degree equations and linear inequalities involving absolute values.	7. Solve and graph the solution on a number line. (a) $\|3x+1\| = 7$ (b) $2 + \|3x+1\| \leq 7$
⑧ Applying linear inequalities to practical problems.	8. Beth has quiz grades of 91, 83, 69, and 74. What is the lowest grade she can receive on a fifth quiz that will enable her to maintain an average of at least 81?

ALGEBRA II B,C

9. Using a calculator to evaluate given polynomials.	9. Use a calculator to find the value of each polynomial, using the given value of the variable. (a) x^2-3x+4; $x = 2.5$ (b) $4x^3+2x-1$; $x = -3.2$
C. LINEAR EQUATIONS	
① Locating points on the Cartesian plane.	1. Graph each ordered pair of numbers. (a) (7,-4) (c) (1.5,4) (b) (-8,0) (d) (-6,-5)
② Solving open sentences in two variables.	2. Find the solution set of each open sentence if each variable represents a whole number. (a) $x+y = 8$ (b) $r-2s = -8$ (c) $5p+q = 30$
③ Graphing the solution of a linear equation.	3. Use a table of values to graph: (a) $2x-3y = 1$ (b) $4x+y = 0$ (c) $\frac{x}{4}-\frac{y}{2} = 1$
④ Changing a linear equation into standard form.	4. Write each equation in standard form. ($Ax+By = C$) (a) $x = \frac{1}{3}y - 2$ (b) $\frac{3}{4}x + y - 1 = 0$ (c) $y = 8x-12$
⑤ Finding the x intercept and the y intercept of a linear equation.	5. Give the: (a) y intercept: $2x-3y = 3y-2$ (b) x intercept: $y = 2x-5$
⑥ Using the definition of slope to calculate the slope of a line, given two points on the line.	6. Calculate the slope of the line passing through: (a) (-7,3) and (4,6) (b) (-8,4) and (-6,-4)

ALGEBRA II C,D

⑦ Developing the equation of a line, given a point on the line and its slope.	7. Find the equation of the line with slope 5 and y intercept $\frac{1}{2}$.
⑧ Developing the equation of a line, given two points on the line.	8. Find the equation of the line passing through (4,6) and (2,-2).
⑨ Changing a linear equation to the y form and identifying the slope and y intercept.	9. Change to the slope intercept form and then give the slope and the y intercept. (a) $3x-6 = x+2y$ (b) $3y-2x = -2y+5$
⑩ Graphing a linear equation by using the slope and the y intercept.	10. Graph the line with a slope of $-\frac{2}{3}$ that crosses the y axis at (0,5).
⑪ Determining whether the graphs of a pair of linear equations are parallel or perpendicular without graphing.	11. Determine without graphing whether these equations represent parallel lines: $2x-1 = y$ $6x+3y = 3$ What must K be so that these equations represent perpendicular lines? $2x-5y = 7$ $Kx+3y = 9$
⑫ Writing the equation of a line that passes through a certain point and runs parallel or perpendicular to a given line.	12. Find the equation of the line passing through: (a) (0,3) and parallel to the line $2x-3y = 7$ (b) (-6,3) and perpendicular to $2x-3y = 6$

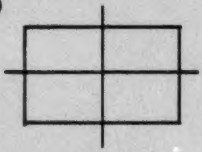
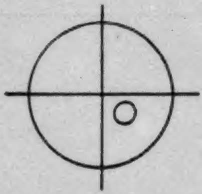
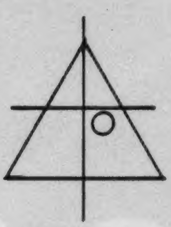
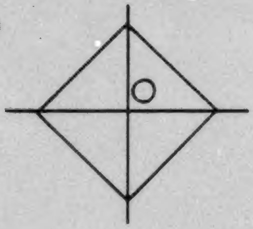
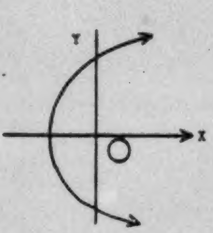
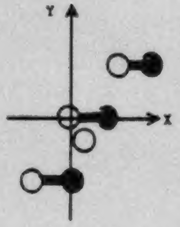
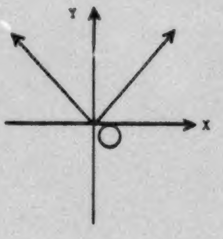
D. POLYNOMIALS AND FACTORING

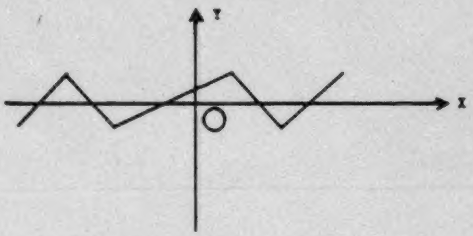
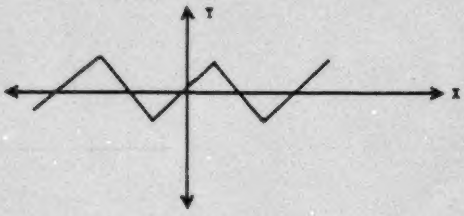
① Identifying the coefficient and the degree of any term in an algebraic expression.	1. $-x^3+3x^2-2+x^4+x^3$ (a) What is the coefficient of the second term? (b) What is the degree of the fourth term?

ALGEBRA II D

② Putting an algebraic expression in increasing or decreasing powers of a variable and classifying the expression according to the number of terms.	2. $-2x^3+4x^2-2+3x^4+2x^3$ (a) Simplify the expression and arrange it in decreasing powers of x. (b) Classify the simplified form of the expression according to the number of terms.
③ Adding, subtracting, and multiplying polynomials.	3. Simplify: (a) $(8x^2-2x+3)+(x-4y-2+2x^2)$ (b) $(5x^2-4y+7)-(3x-2y-4+3x^2)$ (c) $(5x^2)(-2x)^2(-2x^2)$ (d) $3x(x+2)(x^2-x-2)$
4. Using the Binomial Theorem to find the nth power of a binomial.	4. Multiply: (a) $(2x+1)^4$ (b) $(3x-2y)^5$ (c) $(x+2)^6$
5. Using the compound interest formula to solve investment problems.	5. Suppose $3000 is invested at 5%, compounded annually. How much money will be in the account at the end of two years? $A = P(1+r)^t$
⑥ Removing common factors and creating common factors by grouping.	6. Factor completely: (a) $6x^2y-12xy^2+6xy$ (b) $x^2(x+1)+4(x+1)$ (c) $x^4-x^2+4y^2x^2-4y^2$
⑦ Factoring a binomial in the form of the difference between squares and the sum or difference between cubes.	7. Factor completely: (a) $4x^2-49$ (b) $3y^2-75$ (c) y^4-1 (d) $27x^3+8y^3$
⑧ Factoring a quadratic trinomial.	8. Factor completely: (a) $x^2-16x+64$ (b) $4x^6+13x^3+3$ (c) $2y-84+2y^2$ (d) $12x^2-17xy-5y^2$
⑨ Solving equations that factor.	9. Solve: (a) $9x^2+5x = 0$ (b) $3x^2-3 = 0$ (c) $9x^2-12x+4 = 0$ (d) $2x^3+11x^2-21x = 0$

⑩ Translating verbal problems into quadratic equations and solving them.	10. (a) The product of two consecutive odd integers is 195. Find the integers. (b) The hypotenuse of a right triangle is 13 feet. One of the legs is 7 feet longer than the other. Find the length of each leg.
⑪ Solving inequalities by factoring.	11. Find and graph the solution set of (a) $x^2+x > 12$ (b) $t^2-6t+9 \leq 0$
⑫ Using a calculator to find powers of a number.	12. Evaluate the following, using a calculator. (a) $(1.3)^5$ (b) $(.27)^4$ (c) $(1.05)^{\frac{1}{2}}$
E. RELATIONS, FUNCTIONS, TRANSFORMATIONS	
① Listing the domain and range of a given relation.	1. List the domain and range of the relation $\{(-2,0), (-5,1), (2,6), (7,1), (-3,8)\}$
② Graphing a relation that is defined by a sentence in two variables.	2. Graph the relation that is the solution set of $y \geq 3x-5$

3. Identifying figures that are symmetric with respect to a line and a point.	3. Test the following figures for symmetry with respect to the x axis, y axis, origin, and the line y = x. (a)  (b)  (c)  (d) 
4. Testing the equation of a relation for symmetry with respect to the x axis, y axis, origin, and the line y = x.	4. Test the following equations for symmetry with respect to the x axis, y axis, origin, and the line y = x. (a) $x^2+y^2 = 16$ (b) $y^2 = x^2+8$ (c) $y = 2x^3$ (d) $5x+5y = 3$ (e) $y = 2x^2$
⑤ Writing the equation of the inverse of a linear function.	5. Write the equation of the inverse of $y = 3x+2$.
⑥ Determining from its graph whether a relation is a function.	6. Which of the following are graphs of functions?  (a)  (b)  (c)

⑦. Evaluating a function for a given value of x.	7. Given $f(x) = 2x^2-x$, find: (a) $f(3)$ (b) $f(-4)$ (c) $f(a+b)$
8. Sketching vertical or horizontal translations of a given graph.	8. Given the graph of $y = f(x)$ below, sketch the graphs of the following equations.  (a) $y = f(x)+3$ (b) $y = f(x-2)$
9. Sketching stretchings and shrinkings of a given graph.	9. Given the graph of $y = f(x)$ below, sketch the graphs of the following equations.  (a) $y = f(2x)$ (b) $y = f[(-\frac{1}{2})x]$
10. Showing that certain functions can be classified as even or odd.	10. (a) Show that $f(x) = x^2+3x^4$ is an even function. (b) Show that $g(x) = x^5-3x^3$ is an odd function.

F. FRACTIONAL EXPRESSIONS AND EQUATIONS	
① Simplifying rational expressions involving multiplication and division.	1. (a) Multiply and simplify: $\frac{x^2-3x-4}{x^2} \cdot \frac{x^2+2x}{2x-8}$ (b) Divide and simplify: $\frac{x^2+4x+4}{x^2-9x} \div \frac{x^2-4}{x^2+2x-15}$
② Performing the operations of addition and subtraction on rational expressions.	2. (a) Add: $\frac{y+8}{y-5} + \frac{2y}{5-y}$ (b) Add: $\frac{3A+2}{2A^2-A-10} + \frac{8}{2a^2-7A+5}$ (c) Subtract: $\frac{B-9}{B^2-16} - \frac{7-B}{16-B^2}$
③ Simplifying complex expressions.	3. Simplify: $\frac{\frac{1}{x} + \frac{1}{y}}{\frac{x^2 - y^2}{xy}}$
④ Dividing one polynomial by another of lower or equal degree.	4. Divide: $(y^4-y^2-54) \div (y^2-3)$
⑤ Solving equations and inequalities whose terms have fractional coefficients.	5. Solve: (a) $\frac{x^2}{4} - \frac{5x}{6} - \frac{2}{3} = 0$ (b) $\frac{2x-35}{15} < \frac{3x}{10} - 2$
⑥ Solving equations containing fractions.	6. Solve: (a) $-\frac{1}{3} - \frac{5}{4y} = \frac{3}{4} - \frac{1}{6y}$ (b) $\frac{30}{x^2-9} = \frac{5}{x-3} - 1$
⑦ Finding the solutions to verbal problems that use fractional equations.	7. John can mow a lawn in 4 hours. Roy can mow the same lawn in 5 hours. How long would it take both of them, working together, using two lawn mowers, to mow the lawn?

⑧ Solving problems involving direct and indirect variations.	8. (a) The weight x of an object on the moon varies directly as its weight y on the earth. A person who weighs 95 kg. on earth weighs 15.2 kg. on the moon. How much would a 105 kg. person weigh on the moon? (b) The wavelength w of a radio wave varies inversely as its frequency F. A wave with a frequency of 1200 kilocycles per second has a length of 300 meters. What is the length of a wave with a frequency of 800 kilocycles per second?
⑨ Using a calculator to solve direct and indirect variation (proportion) problems.	9. Use a calculator to solve the following proportion for x, correct to the nearest thousandth: $\frac{x}{141} = \frac{1.5627}{233}$
G. EXPONENTS, POWERS, AND ROOTS	
① Demonstrating the laws of exponents in the simplification of algebraic expressions.	1. Simplify (use positive exponents): (a) $(-3x^3y^4)^3$ (b) $(x^2+y)^0$ (c) $\frac{(5x^2y-2)^{-3}}{3z}$ (d) $\frac{(x^2y^3)^3 \cdot (3x^{-2}y)^2}{(xy)^{-3}}$
② Converting between scientific and decimal notation.	2. (a) Convert 92,000,000,000 to scientific notation. (b) Convert 2.066×10^{-12} to decimal notation.
③ Identifying roots of real numbers.	3. Evaluate each radical. If the radical does not represent a real number, so state. (a) $\sqrt{121}$ (b) $\sqrt[4]{256}$ (c) $\sqrt{-64}$

ALGEBRA II G

④ Simplifying radicals (including rationalizing the denominator). Any variables used take only positive values.	4. Simplify: (a) $\sqrt[3]{16a^5}$ (b) $\sqrt{\frac{12}{5a}}$ (c) $\sqrt[4]{\frac{s^2}{2t^3}}$
⑤ Performing the operations of addition, subtraction, multiplication, and division on radical expressions.	5. Perform the indicated operations and simplify: (a) $\sqrt{20} \cdot \sqrt{15}$ (b) $\sqrt{72xy} \div 2\sqrt{2}$ (c) $5\sqrt{50} - 2\sqrt{18} + 9\sqrt{32}$
⑥ Converting between radical and exponential forms of an expression.	6. (a) Convert to radical form: (i) $6^{\frac{1}{3}}$ (ii) $(xy^2)^{\frac{2}{5}}$ (b) Convert to exponential form: (i) $\sqrt[5]{6^2}$ (ii) $\sqrt[4]{2x^2y^3}$
⑦ Using fractional exponents in the simplification of radical expressions. Any variables used take only positive values.	7. Use fractional exponents to simplify: $\sqrt[12]{64x^6y^6}$
⑧ Solving equations that contain radicals.	8. Solve: (a) $\sqrt{3x-2} = 3$ (b) $\sqrt{2x-3} = \sqrt{x+7} - 2$
⑨ Converting decimal representation to rational form.	9. Write as a common fraction in the simplest form: (a) $3.\overline{21}$ (b) 0.4375 (c) $0.\overline{711}$

⑩ Using a calculator to evaluate radicals.	10. Use a calculator to evaluate each radical to four decimal places. (a) $\sqrt[3]{12}$ (b) $\sqrt[7]{930}$ (c) $\sqrt[4]{2^6}$

H. COMPLEX NUMBERS

① Simplifying expressions containing complex numbers.	1. Write in the "i" form, where $i^2 = -1$. (a) $\sqrt{-25}$ (c) i^{20} (b) $-\sqrt{-12}$ (d) $\sqrt{(-1)^2}$
② Adding, subtracting, multiplying, and dividing complex numbers.	2. Write in the form a + bi: (a) 3i + (8-5i) (b) (2+3i)(3+4i) (c) $(-3-3i)^2$ (d) $\frac{3+2i}{2+i}$
③ Solving equations involving complex numbers.	3. Solve for x and y: $-3x - yi = 10 + 7i$
④ Finding an equation having specified complex numbers as roots.	4. Find an equation having the solutions i, 2i, and -i.
⑤ Graphing a complex number.	5. Graph -2-2i and 4+3i.
6. Finding the absolute value of a complex number.	6. Find the absolute value of (a) -3-i (b) 4-3i

I. QUADRATIC EQUATIONS AND QUADRATIC FUNCTIONS

① Solving quadratic equations by completing the square.	1. Solve by completing the square. (a) $x^2-8x+6 = 0$ (b) $x^2+4x = 7$ (c) $x^2+3x+4 = 0$
② Solving quadratic equations by factoring and by using the quadratic formula.	2. Solve by factoring and/or by using the quadratic formula. (a) $3x^2+2x-4 = 0$ (b) $5t^2+12t+8 = 0$ (c) $8x^2 = 14x+15$
③ Extending quadratic equations to the solution of verbal problems.	3. A garden is twice as long as it is wide. The area of the garden is $200m^2$. Find the dimensions of the garden. $(A = L \cdot W)$
④ Determining the value of the discriminant of a quadratic equation and using it to state the nature of the roots of that equation.	4. Determine the value of the discriminant and state the nature of the roots. (a) $x^2-12x+36 = 0$ (b) $2x^2 = -5$
⑤ Solving formulas that involve quadratic equations for a given letter.	5. Solve for d: $K = \frac{1}{3} Fd^2$
⑥ Utilizing formulas to solve interest and motion problems.	6. An object is dropped from a height of 2304 ft. How long will it take to reach the ground? $(s = 16t^2 + V_0t)$
⑦ Using substitution to solve equations that are quadratic in form.	7. Solve, using substitution: $(x^2+2)^2-21(x^2+2)+54 = 0$
⑧ Graphing a quadratic function in standard form and determining the vertex, axis of symmetry, and the maximum or minimum value.	8. Graph the following quadratic functions, stating the vertex, axis of symmetry, and the maximum or minimum value: (a) $f(x) = -4x^2$ (b) $f(x) = 3(x-1)^2$ (c) $f(x) = -2(x+5)^2-3$

⑨ Converting a quadratic function to standard form and determining the vertex, axis of symmetry, and the maximum or minimum value.	9. Convert the following quadratic functions to standard form and determine the vertex, axis of symmetry, and the maximum or minimum value: (a) $f(x) = x^2-6x+8$ (b) $f(x) = -2x^2+2x+3$
⑩ Finding the x intercept(s) of the graph of a quadratic function.	10. Find the x intercepts of the following, if they exist: (a) $f(x) = x^2-2x-5$ (b) $f(x) = 2x^2-3x+6$
⑪ Solving problems involving quadratic functions.	11. What are the dimensions of the largest rectangular pen that can be enclosed with 100 meters of fence?
J. SYSTEMS OF EQUATIONS AND SYSTEMS OF INEQUALITIES	
① Solving a system of linear equations graphically.	1. Solve graphically: (a) $2x+3y = -8$ $x-4y = 7$ (b) $2y = 6-x$ $3x-2y = 6$
② Solving a system of linear equations by substitution.	2. Solve by substitution: (a) $4x+y = 7$ $3x-2y = 8$ (b) $2x-y = 6$ $3x+5y = 22$
③ Solving a system of linear equations by addition.	3. Solve by addition: (a) $x-4y = 7$ $3x+5y = 14$ (b) $2x-3y = 0$ $-4x+3y = -1$
④ Using pairs of linear equations in two variables to solve problems.	4. For delivering newspapers, a delivery service charges a fixed amount for the first 50 g and a fixed rate for each additional 10 g. If the charge for a 150 g newspaper is 72 cents and that for a 220 g newspaper is one dollar, what are the rates?

⑤ Graphing linear inequalities in two variables.	5. Graph on a plane: (a) $6-2y \leq -3x$ (b) $y < -\frac{1}{2}x-4$
⑥ Graphing systems of linear inequalities in two variables.	6. Graph on a plane: (a) $y \leq 2x$ $y > -x+3$ (b) $2y-x \leq 2$ $y-3x \geq -1$
⑦ Solving a system of equations in three variables.	7. Solve: (a) $\begin{bmatrix} 2x-y+3z = 9 \\ x+y+2z = 2 \\ 3x-2y-z = 9 \end{bmatrix}$ (b) $\begin{bmatrix} 4x-2y-z = 19 \\ x-3y+z = 7 \\ 3x+y-2z = 12 \end{bmatrix}$

K. POLYNOMIAL FUNCTIONS

① Determining whether a given number is a zero of a polynomial P(x).	1. Determine whether the following numbers are zeros of the polynomial $P(x) = x^2-4x-21$. (a) 7 (b) 3 (c) -3
② Using synthetic division to find the quotient and remainder in division of polynomials.	2. Use synthetic division to find the quotient and remainder in (a) $(x^3-5x^2+4x+7) \div (x-1)$ (b) $(12x^4+17x^3+8x-40) \div (x+2)$ (c) $(2x^{12}-x^9-4) \div (x+1)$
③ Using the Remainder Thm. to find the remainder when a polynomial is divided by x-r.	3. Use the Remainder Thm. to find the remainder when $2x^3-3x^2+x-1$ is divided by $x-2$.
④ Using the Factor Thm. to determine whether (x-r) is a factor of a given polynomial.	4. Use the Factor Thm. to determine whether $(x-2)$ is a factor of x^3+6x^2-x-30.
⑤ Using synthetic division to evaluate a polynomial P(x).	5. Use synthetic division to find the value of the function at 2 if $P(x) = 2x^5-3x^3+4x^2-3x+5$.

⑥ Finding the zeros of a polynomial P(x).	6. Given $P(x) = x^4-16$, find the zeros of P(x).
⑦ Writing a polynomial P(x), given the zeros of the polynomial.	7. Find a polynomial of degree 4 with $3i$, $-3i$, $\sqrt{5}$, and $-\sqrt{5}$ as zeros of the polynomial.
⑧ Listing the possible rational roots of a polynomial and checking them by using synthetic division.	8. Given $P(x) = 3x^4-5x^3-5x^2-19x-6$, list the possible rational roots and check them by using synthetic substitution.
⑨ Using synthetic substitution to factor a polynomial.	9. Factor the polynomial $x^4+x^3-13x^2-x+12$ completely over the rational numbers, using synthetic substitution.
L. EQUATIONS OF SECOND DEGREE	
① Finding the distance between any two points in a plane, using the distance formula.	1. Find the distance between the points. (a) (3,5), (5,8) (b) (1,-3), (2,-5) (c) (m,n), (n,m)
② Stating the coordinates of the midpoint of a segment, given the end points.	2. Find the midpoint of a segment, given the end points. (a) $(\frac{1}{2},7)$, $(\frac{1}{2},5)$ (b) $(2\sqrt{5},1)$, $(-\sqrt{5},3)$ (c) (-A,B), (A,B)
③ Using the equation of a circle to find the center and radius of a circle.	3. Find the center and radius of each circle. (a) $x^2+y^2 = 5$ (b) $(x-4)^2+y^2 = \frac{1}{16}$ (b) $(x-4)^2+y^2 =$ (c) $x^2+y^2-8x+4y-5 = 0$

④ Finding the equation of a circle, given the center and radius of the circle.	4. Find the equation of a circle with center and radius as given: (a) center (-3,7) radius $\sqrt{3}$ (b) center (-5,-7) radius $\frac{1}{2}\sqrt{3}$
⑤ Graphing an ellipse, recognizing the center, vertexes, and foci of the ellipse.	5. For each ellipse, find the center, vertexes, and foci, then graph the ellipse. (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (b) $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{9} = 1$
⑥ Graphing a hyperbola, recognizing the center, vertexes, foci, and asymptotes of the hyperbola.	6. For each hyperbola find the center, vertexes, foci, and asymptotes, then graph the hyperbola. (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (b) $\frac{(x-1)^2}{16} - \frac{(y-2)^2}{9} = 1$
⑦ Graphing a parabola, recognizing the vertex, focus, and directrix of the parabola.	7. For each parabola, find the vertex, focus, and directrix, then graph the parabola. (a) $x^2 = 2y$ (b) $y^2 = -12x$
⑧ Solving quadratic systems by graphing.	8. Use graphing to solve: (a) $\begin{cases} x^2+y^2 = 10 \\ x-3y+10 = 0 \end{cases}$ (b) $\begin{cases} x^2-y^2+4 = 0 \\ x+y = 1 \end{cases}$ (c) $\begin{cases} x^2+y^2 \le 4 \\ y \ge x^2-1 \end{cases}$
⑨ Solving quadratic systems by algebraic methods.	9. Use algebraic methods to find solutions for these quadratic systems. (a) $x^2+4y^2 = 25$ $x+2y = 1$ (b) $x^2+4y^2 = 49$ $2x^2-3y^2 = -12$

M. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

① Defining and drawing the graph of the exponential and the logarithmic functions.	1. Graph: (a) $y = 2^x$ (b) $y = \log_{\frac{1}{2}} x$
② Converting between exponential and logarithmic equations.	2. (a) Change $\log_y x = z$ to exponential form. (b) Change $z^y = x$ to logarithmic form.
③ Solving simple logarithmic equations.	3. Solve for x: (a) $\log_2 x = -3$ (b) $\log_4 x = \frac{1}{2}$ (c) $\log_{10} x = 4$ (d) $\log_x 81 = 4$
④ Using the properties of logarithms to rewrite and to simplify given expressions.	4. Given $\log 5 = x$, $\log 6 = y$ and $\log 2 = z$, find: (a) $\log 30$ (b) $\log 3$ (c) $\log 6^5$ (d) $\log \sqrt{5}$
⑤ Using the table to find common logarithms and antilogarithms.	5. Use the table in your text to find the value of: (a) log 83,000 (b) log .00842 (c) $10^{3.4742}$ (d) Antilog (9.1987-10)

⑥ Solving exponential and logarithmic equations.	6. Solve for x: (a) $9 = 3^{3x-2}$ (b) $\log_9 x = \frac{1}{2}$ (c) $\log x + \log 4 = 2$ (d) $2 \log_4 x = \log_4 9$ (e) $2^x = 3$
⑦ Solving verbal problems that require the use of exponential and logarithmic equations.	7. The amount A that principal P will be worth after t years at interest rate r, compounded annually, is given by the formula $A = P(1+r)^t$. Suppose $5000 principal is invested at 8% interest and yields $8569. How many years was it invested?
⑧ Using a calculator to solve exponential and logarithmic equations.	8. Use a calculator to solve the following: (a) $5^t = 10$ (b) $Z^{1.8} = 3.6$
N. SEQUENCES AND SERIES	
① Listing n terms or the nth term of any sequence, given the general term.	1. (a) List the first five terms of the sequence whose general term is n^2-n+7. (b) Find the third term of n^n+1.
② Finding the rule that generates a given sequence.	2. Find the general term of the sequence $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$
③ Listing n terms of any series, given the general term.	3. Rewrite $\sum_{k=1}^{6} 2^{k-1}$ without using sigma notation.
④ Finding the rule that generates a given series and rewriting the series in sigma notation.	4. Rewrite 4+8+12+16 using sigma notation.

ALGEBRA II N

5. Recognizing a sequence as arithmetic and finding a_1, n, d, or the nth term using $a_n = a_1 + (n-1)d$.	5. (a) Write the rule that can be used to find the nth term of an arithmetic sequence. (b) Given $a_1 = 5$, $d = 4$, find a_7. (c) Given $a_7 = 8$, $d = \frac{1}{2}$, find a_1.
⑥ Finding the sum of n terms of an arithmetic series.	6. (a) Write the rule that can be used to find the sum of n terms of an arithmetic series. (b) Find the sum S_n of an arithmetic series with $a_1 = 2$, $d = -3$ and $n = 20$. (c) Find $\sum_{K=1}^{20} (2K+5)$
⑦ Recognizing a sequence as geometric and finding a_1, r, n, or the nth term, using $a_n = a_1 r^{n-1}$.	7. (a) Write the rule that can be used to find the nth term of a geometric sequence. (b) Given the sequence 16, -8, 4, -2, ... , find the common ratio, r. (c) Given $a_6 = 6$ and $r = 2$, find a_1.
⑧ Finding the sum of n terms of a geometric series.	8. (a) Write the rule that can be used to find the sum of the first n terms of a geometric series. (b) Find a_n and s_n, given $a_1 = 2$, $r = 3$, and $n = 4$.
⑨ Finding the sum of an infinite geometric series.	9. Determine if this infinite geometric sequence has a sum. 8, 4, 2, . . ., 2^{4-n}
10. Using a calculator for work with sequences and series.	10. Use a calculator to find (a) 25! (b) Which is greater. $5^{5!}$ or $5!^{5}$? (c) $\sum_{n=1}^{10} n!$

O. MATRIXES AND DETERMINANTS

1. Solving systems of linear equations in two or three variables, using matrixes.	1. Solve, using matrixes: (a) $\begin{bmatrix} x-2y = 5 \\ 3x+4y = -5 \end{bmatrix}$ (b) $\begin{bmatrix} x+2y+z = 5 \\ 2x-y+z = 4 \\ 3x+y+4z = 1 \end{bmatrix}$
2. Evaluating determinants of the second and third order.	2. Evaluate the given determinant. (a) $\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & 1 \\ 2 & 5 & -1 \\ 1 & 1 & -12 \end{bmatrix}$
3. Using Cramer's Rule to solve systems of equations in two or three variables.	3. Solve, using Cramer's Rule: (a) $\begin{bmatrix} 5x+2 = 3y \\ 4x+2y-5 = 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2x+y-z = -4 \\ x+5y-z = 2 \\ 2x+3y+z = 0 \end{bmatrix}$
4. Performing the operations of addition, subtraction, and multiplication of matrixes of the same dimensions.	4. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 2 \\ -1 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ Find: (a) A+B (c) AB (b) A-B (d) 3B

P. COMBINATORIAL ALGEBRA: PROBABILITY

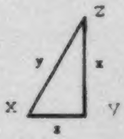
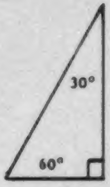
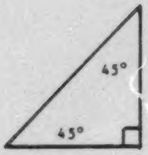
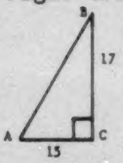
1. Finding the total number of permutations, given n objects taken n at a time, and listing the permutations.	1. (a) How many permutations of the set {a,b,c} taken three at a time are there? List them. (b) How many permutations of the set {a,b,c,d} taken four at a time are there? List them.

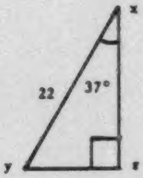
2. Finding the total number of permutations, given n objects taken r at a time, and listing the permutations.	2. (a) How many permutations of the set {a,b,c,d} taken two at a time are there? List them. (b) How many permutations of the set {a,b,c,d} taken three at a time are there? List them.
3. Evaluating factorial and permutation notation.	3. Evaluate: (a) 5! (b) 7! (c) ${}_6P_6$ (d) ${}_6P_4$
4. Finding the number of combinations of n objects taken r at a time.	4. Evaluate: (a) ${}_4C_3$ (b) $\binom{10}{8}$
5. Applying combinational algebra to realistic problems.	5. (a) A class wants to select a president, vice-president, and treasurer from six members. How many possible slates of officers are there? (b) If five cards are picked at random from a deck of fifty-two cards, how many different hands are possible?

Q. ALGEBRA II AND CAREERS

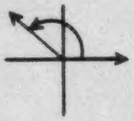
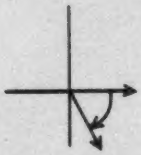
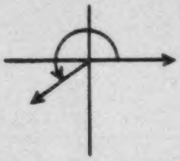
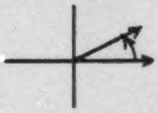
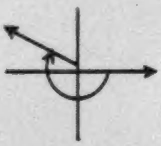
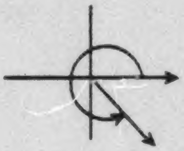
Investigating one job from each of three occupational clusters in which skills learned in Algebra II can be applied.

TRIGONOMETRY

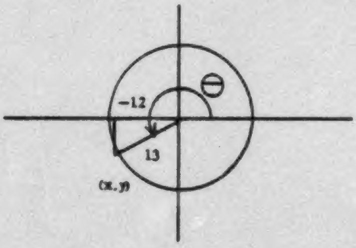
A. INTRODUCTION TO TRIGONOMETRY	
① Naming the sides of a right triangle.	1. Given a right triangle ABC with the right angle located at C, identify the hypotenuse and the legs of the triangle.
② Determining the lengths of the sides of a right triangle, using the Pythagorean theorem.	2. In a right triangle ABC with the right angle at C, side a = 12, and side c = 13. Find the length of side b.
③ Naming the ratio of sides in a right triangle by the sine, cosine, and tangent functions.	3. Given right triangle XYZ with the right angle at Y, which trigonometric functions of angle X are given by the ratios $\frac{x}{y}, \frac{z}{y}, \frac{x}{z}$? 
④ Finding the values of the sine, cosine, and tangent functions in the 30°, 60°, 90° triangle and the 45°, 45°, 90° right triangle.	4. Give the value of (a) sine 30° (b) cos 60° (c) tan 45° (d) cos 45°  
5. Determining the sine, cosine, and tangent ratios, using a calculator.	5. Using a calculator, find the following: (a) sine 52° (b) cos 36° (c) tan 85°
⑥ Reading trigonometric tables to determine the sine, cosine, and tangent functions.	6. Use trigonometric tables to find the following: (a) sin 36° (b) cos 54° (c) tan 47°
⑦ Solving right triangles, given two sides or given an angle and a side.	7. Solve the following right triangles: (a) Find the measure of the angles. 

	(b) Find the lengths of the two sides. 
⑧ Solving word problems involving right triangles through the use of trigonometry.	8. A twenty-foot ladder makes an angle of 65° with the street while resting against the side of a house. How far up the side of the house does the ladder reach?
9. Solving the trigonometry equations of a right triangle, using a calculator.	9. Solve, using a calculator: $\sin 38^\circ = \frac{22}{x}$

B. ANGLE MEASUREMENT

① Measuring angles in degrees and angles in standard position.	1. Match the angles and measurement in degrees.  (a)  (b)  (c)  (d)  (e)  (f) (i) 275°, (ii) -60°, (iii) 330°, (iv) 375°, (v) 105°, (vi) -200°

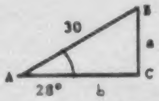
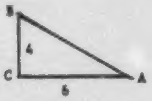
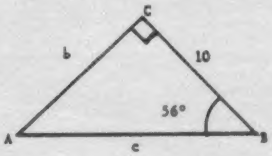
TRIGONOMETRY B

② Measuring angles in radians and angles in standard position.	2. Complete the following: Angle Measurement Degrees = Radians 180° = ___ ___° = $\frac{3\pi}{4}$ ___° = $\frac{\pi}{3}$ ___° = $\frac{3\pi}{2}$ 45° = ___
③ Finding the sine and cosine of angles that are in standard position.	3. If θ is a third-quadrant angle and $\cos\theta = -\frac{12}{13}$, find $\sin\theta$. 
④ Using tables to find the sine and cosine of first-quadrant angles.	4. Use tables to determine: (a) $\cos 48^\circ 18'$ (b) $\sin 29^\circ 36'$ (c) θ if $\sin\theta = 0.7660$ (d) θ if $\cos\theta = 0.9063$
⑤ Using tables and reference angles to find the sine and cosine of any angle.	5. Determine the reference angle and then use tables to find: (a) $\cos 144^\circ$ (b) $\sin 215.6^\circ$ (c) θ if $\sin\theta = -0.2924$ and θ is in the third quadrant (d) θ if $\cos\theta = 0.9774$ and θ is in the fourth quadrant
⑥ Graphing the sine and cosine functions.	6. Sketch the graph for two periods of these functions: (a) $y = 3\sin x$ (b) $y = \cos 2x$

TRIGONOMETRY B,C

7. Using the tangent, cotangent, secant, and cosecant functions.	7. Evaluate without using tables or a calculator: (a) $\cot 300^\circ =$ ____ (b) $\sec \pi =$ ____ (c) $\tan 135^\circ =$ ____ (d) $\csc \frac{2\pi}{3} =$ ____
8. Simplifying expressions using trigonometric identities.	8. Simplify: (a) $\tan \theta \sec \theta \sin \theta =$ (b) $\frac{1}{\cos^2\theta} - \frac{1}{\cot^2\theta} =$ (c) $\frac{\sin x \cos x}{1-\cos^2 x} =$
9. Using a calculator to find trigonometric functions.	9. Use a calculator to find the following: (a) $\tan 200^\circ$ (b) $\csc 307^\circ 40'$ (c) θ when $\sec \theta = 2.2$ (d) θ when $\csc \theta = -\frac{5}{4}$

C. SOLUTIONS OF TRIANGLES

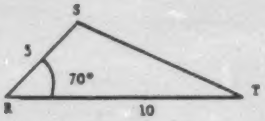
1. Finding the side and angles of a right triangle.	1. Find the indicated angle or side in these right triangles. (a)  Find side b (b)  Find the measure of angle A (c)  Find side b

TRIGONOMETRY C

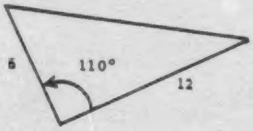
② Finding the area of any triangle when the lengths of two sides and the measure of the included angle are known.

2. Find the area of these triangles.

(a)



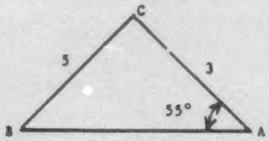
(b)



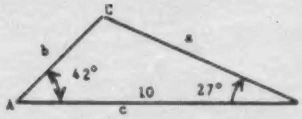
③ Using the law of sines to find a certain angle or side.

3. Find the indicated side or angle.

(a) ∠B = ?



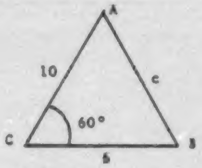
(b) b = ?


C
b
a
42°
10
27°
A
c

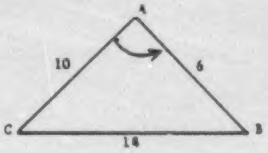
④ Using the law of cosines to find a certain angle or side.

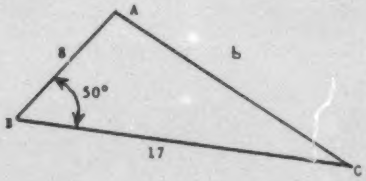
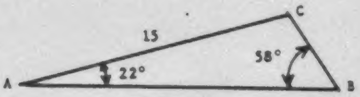
4. Find the indicated side or angle.

(a) c = ?



(b) ∠A = ?



⑤ Solving any triangle for all angle measures and all lengths of sides.	5. Solve these triangles by finding all the missing angle measures and lengths of sides. (a)  (b) 
6. Finding the sum of two vectors.	6. Find the sum (resultant) of these vectors. (a) a force of 9T up and a force of 5T to the left (b) an airplane velocity of 650 km/h north and a wind blowing at 50 km/h from east to west
7. Finding the horizontal and vertical components of a vector.	7. Find the x and y components of a vector 20 units long that has a direction angle of 130°.
D. IDENTITIES, EQUATIONS, AND TRIGONOMETRIC FUNCTIONS OF COMPLEX NUMBERS	
① Using the sum or difference identities for the sine and cosine functions.	1. Use sine and cosine sums or difference identities to solve the following: (a) $\cos 75^\circ =$ _____ (b) $\sin 165^\circ =$ _____ (c) $\cos 105^\circ =$ _____
② Using the sum or difference identities of the tangent function.	2. Use the tangent sum or difference identities to simplify the following: (a) $\frac{\tan 50^\circ + \tan 5^\circ}{1-\tan 50^\circ \tan 5^\circ}$ (b) $\frac{\tan 130^\circ - \tan 15^\circ}{1+\tan 130^\circ \tan 15^\circ}$

TRIGONOMETRY D

③ Using the double-angle formulas.	3. Use the double-angle formulas to simplify the following: (a) $2\sin\alpha\ \cos\alpha$ (b) $1-2\sin^2\theta$ (c) $\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$
④ Using the half-angle formulas.	4. Use the half-angle formulas to find the exact values of (a) $\sin\frac{5\pi}{8}$ and $\cos\frac{5\pi}{8}$ (b) sine, cosine, and tangent of 105°
⑤ Graphing the tangent, cotangent, secant, and cosecant functions.	5. Sketch the graph of these functions over the interval $0 \leq x < 2\pi$ (a) $y = \tan 2x$ (b) $y = 3\csc x$ (c) $y = -\cot x$
6. Using the inverse sine and inverse cosine functions to evaluate given expressions.	6. Evaluate each of these expressions: (a) $\sin^{-1}(\sin\frac{3\pi}{2})$ (b) $\cos^{-1}(\cos\frac{5\pi}{3})$ (c) $\sin^{-1}[\cos(-\frac{\pi}{6})]$
⑦ Using the inverse sine, cosine, tangent, cotangent, secant, and cosecant functions to evaluate expressions.	7. Evaluate each of these expressions: (a) $\tan(\cot^{-1}5)$ (b) $\sin(\cot^{-1}(-2))$ (c) $\tan(\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{5})$
⑧ Solving trigonometric equations.	8. Solve for θ, $0° \leq \theta < 360°$ (a) $5\cos\theta = -2$ (b) $3\sin\theta + 2\cos\theta = 0$ (c) $5\tan^2\theta - \tan\theta = 0$

TRIGONOMETRY D,E

⑨ Proving trigonometric identities.	9. Prove the following identities: (a) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$ (b) $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$ (c) $\cot 2\theta = \frac{\cot^2\theta-1}{2\cot\theta}$
10. Plotting complex numbers in the complex plane.	10. Plot each set of numbers in the complex plane. (a) $5+3i$, $-4+2i$, $2i$ (b) -2, $-4+3i$, $-1-6i$ (c) $-3-5i$, $-3+5i$, $3+5i$, $3-5i$
11. Expressing a given complex number in both the a+bi form and the trigonometric form.	11. (a) Express $4(\cos 30^\circ + i \sin 30^\circ)$ in the form $a+bi$. (b) Express $2\sqrt{3} - 2i$ in trigonometric form.

E. TRIGONOMETRY AND CAREERS

Investigating one job from each of three occupational clusters in which the skills learned in Trigonometry can be applied.

ADVANCED MATHEMATICS OBJECTIVES

Introduction

Students entering Advanced Mathematics have just completed the Algebra I-Geometry-Algebra II/Trigonometry sequence. One of our first concerns must be to show students that all these "building blocks" fit together; that these seemingly separate subject areas are interwoven and interdependent; that there is unity and continuity in mathematics. Various mathematics organizations, concerned with secondary school mathematics curricula and the present demands of college preparation, have made great efforts to bring about a natural integration of algebra and geometry. The Advanced Mathematics Objectives attempt as much as possible to integrate these strands.

The total list of Advanced Mathematics Objectives presents a challenging goal for the most advanced students. Some students will need more time than others to reach the objectives illustrated by the more demanding examples. Hence, what will be accomplished in one semester will vary from class to class, with the top students completing all topics by the end of the year. A document of this type must have a great deal of flexibility built in, so that it can serve students of different levels of interest, ability, and preparation.

The objectives of Section A (Introductory Chapter) are designed to build a base of understanding and preparation, to give students an overview of some of the larger ideas (like the extension of the number system from counting to complex numbers), and to build students' confidence for the sections that lie ahead.

As students get into the main body of the objectives, they will find that there are topics which they have studied previously. However, these topics are presented on a higher level, and will strengthen students' preparation and deepen their understanding.

Calculators must be available to all students, and teachers must help students to use them wisely and with restraint. Computer applications of Advanced Mathematics should be acquired in a separate course; there is a great deal to be covered in Advanced Mathematics, and time is of the essence.

Every effort must be made to emphasize problem solving applications (e.g., maximizing profits, while minimizing cost; radioactive decay; population growth; etc.). Some of these problems are covered in most textbooks, linked to computer solutions, and lend themselves to individual projects. When students complete such projects, they can be of great value to their classmates.

When we ask students to solve problems (for instance, systems of equations or quadratic equations), we should not ask them to use a particular method; rather, we should ask students to decide for themselves the most efficient approach. They must come to realize that some problems are time-consuming and require detailed analysis; perhaps looking up a forgotten skill, while others can be answered speedily without any loss of accuracy. A critical goal in Advanced Mathematics is developing students' powers of judgment and independent thinking. These attributes are basic to their future success.

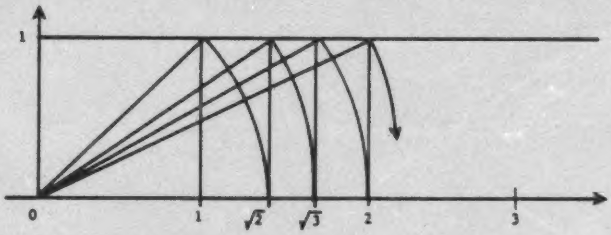
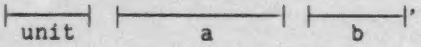
The Advanced Mathematics Objectives are divided into the following sections:

A. Introductory Chapter - Surveying and Checking
B. Relations and Functions
C. Composition and Inverses
D. Trigonometric and Circular Functions
E. Conic Sections
F. Graphs of Polar and Parametric Equations
G. Limits of Functions
H. Polynomial Functions
I. Exponential and Logarithmic Functions
J. Sequences - Series - Limits
K. Permutations and Combinations
L. Binomial Theorem
M. Mathematical Induction
N. Probability
O. Vectors
P. Complex Numbers
Q. Practice
R. Advanced Mathematics and Careers

ADVANCED MATHEMATICS

A. INTRODUCTORY CHAPTER: SURVEYING AND CHECKING

1. Reviewing and obtaining a clearer understanding of prior concepts and their relationships. How and why go from one number system to another?	1. The counting numbers were extended to include zero; (a) next we introduced the integers, etc. $x + 3 = 5$, has a solution in N; $x + 5 = 3$, does not. $3x = 9$, but not $9x = 3$, has a solution in N. Give examples that lead to the necessary extensions to Q, R, and finally C. (b) Do all properties of N hold in Q? Do all properties of Q hold for the irrational numbers? the complex numbers? (c) Complete each of the following and draw a conclusion. $\sqrt{3}\ \sqrt{12} =$; $(3+2i)(3-2i) =$; where $i^2 = -1$; $(3+2i)(3-2i)^{*} =$; $\sqrt{5}\ \sqrt{3} =$; $\sqrt{-5}\ \sqrt{-3} =$; $\sqrt{(-5)(-3)} =$.
2. Judging, with increasing facility, which steps to take first and how they illustrate the field axioms.	2. Give answers in simplest form: (a) $\frac{4}{7} \cdot \frac{21}{20} = \frac{4 \cdot 7 \cdot 3}{4 \cdot 7 \cdot 5} = ?$ (b) $\frac{x^3-x-6}{x^2-4} \cdot \frac{(x+2)^2}{x^2-x-6} = ?$
3. Choosing the most efficient method of solving a problem, without a calculator.	3. Which method would you choose to solve for (x,y)? (a) $\begin{cases} 3x-2y = 12 \\ 5x+y = 7 \end{cases}$ (b) $\begin{cases} ax-5y = b \\ 3x+cy = d \end{cases}$ (c) $\begin{cases} (x-2)^2+(y-3)^2 = 25 \\ 5x + y + 9 = 0 \end{cases}$

	(d) Evaluate for $x = -2$: $y = 3x^5+14x^2-3x+34$, and $y = x^8-100$.
4. Looking at algebraic problems "geometrically."	4. We can give only a rational approximation of an irrational number; yet, there is a "geometrical equivalent." (a) Explain the following ruler and compass construction:  (b) Use ruler and compass only and the given segments:  to construct segments RS, x, and y. (1) $RS = \lvert 2a-b \rvert$ (2) $x = ab$ (3) $y = \sqrt{a}$.
5. Understanding and applying, with proper restrictions, if $a = b$, then $ac = bc$, and if $z \neq 0$ and $yz = xz$, then $y = x$, in solving equations.	5. Are these two equations equivalent? (a) $3x^3+x^2-4x = 0$, and $3x^2+x-4 = 0$. (b) $\tan \theta + 2 \cos \theta = \sec \theta$, and $\sin \theta + 2 \cos^2 \theta = 1$.
6. Realizing that an equation is not solved until it is checked: - by inspection - by complete and careful substitution, if necessary - by use of a calculator, if desirable Understanding whether the steps are reversible; whether imperative and restrictions are variable too.	6. Solve for $x \in R$: (a) $\frac{x+2}{x^2-x-6} = 3 - \frac{4}{x-3}$; (b) $\sqrt{x+1} - \sqrt{x-1} = 1$; (c) $\sqrt{x+1} + \sqrt{x-1} = 1$.

7. Realizing the importance of solving a quadratic equation by factoring or by using the quadratic formula, and the many "disguised" equations one can solve if one recognizes how to group terms or how to substitute.	7. Solve the following equations in C: (a) $x^4-15x^2-16 = 0$; (b) $(x^2+2x)^2-3(x^2+2x+10) = 150$; (c) $(x-3)^2 + \sqrt{(x-3)^2+5} = 7$; (d) $4^{x+3} + 2^{x+2} = 36 \cdot 2^x$; (e) $6(x^2 + \frac{1}{x^2}) - 35(x + \frac{1}{x}) + 62 = 0$.
8. Writing the quadratic equation, given the roots, deriving the root coefficient relation for any quadratic equation, and using to check results. Extending to cubic equations.	8. Write the roots of (a) $ax^2+bx+c = 0, a,b,c\neq0$, r_1 and r_2, and in terms of a,b,c; express in simplified form, r_1+r_2, and r_1r_2 . (b) Solve the equation $x^2-7x-8 = 0$ and find the sum and product of the roots. Compare with the coefficients of the equation and your answers to (a) above. (c) Solve the equation $2x^3+x^2-7x-6 = 0$. Let the roots be r,s,q, and find the value of r+s+q, rs+rq+sq, and rsq. Compare these results with the coefficients of the equation. (d) Generalize for $ax^3+bx^2+cx+d = 0$ and its roots r,s, and q, by giving the value of $-\frac{b}{a} =$, $a\neq0$ $\frac{c}{a} =$, $-\frac{d}{a} =$, from (c) above. (e) Expand and write in descending powers of x, $(x-r)(x-s)(x-q) = 0$.

9. Recognizing which functions have inverses and how to restrict the domain and/or range so that a function has an inverse; finding an expression for f^{-1}.

9. (a) Which of the following have inverses that are functions?

(1) $y = mx+b,\ m \neq 0$;

(2) $y = x^2$;

(3) $y = x^2$ and $x \geq 0$;

(4) $y = x^2$ and $x \leq 0$;

(5) $x^2+y^2 = r^2$ and $y \geq 0$.

(6) $x^2+y^2 = r^2$ and $x \geq 0$ and $y \geq 0$.

(b) Sketch the graph of each of the above and the inverse, if it exists.

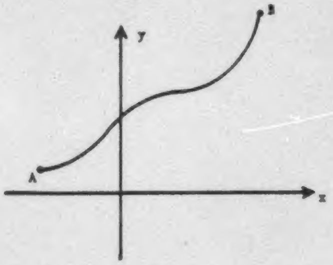
(c) Write the equation for each inverse function above with appropriate restrictions.

(d) For what value of k will $y = kx+b$ be its own inverse?

(e) Give the x-intercept and the y-intercept of the inverse function of

$$\frac{x}{8} + \frac{y}{-2} = 1 \ .$$

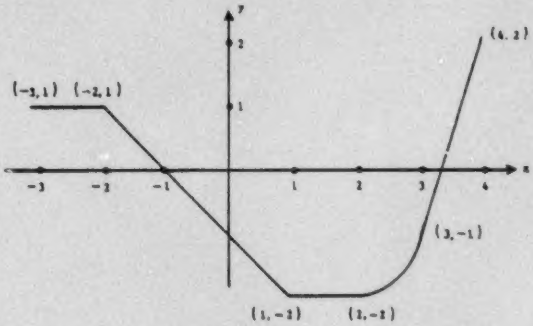
(f) Given the graph to the right of f, draw the graph of f^{-1}.



ADVANCED MATHEMATICS A

10. Evaluating expressions involving absolute values, solving equations with absolute values, and drawing graphs of functions with absolute values.	10. Find all x such that (a) $\|x-5\| \leq 3$; (b) $-3 < \|x-2\| \leq 0$; (c) $\|x\|^2 - 2\|x\| - 3 = 0$. (d) Draw the graph of $y = 2\|x-1\| - 3$ and $y = 2(x-1)^2-3$. (e) Draw the graph of $y = \|x^2-2x-3\|$.
11. Using the discriminant of a quadratic equation in problems.	11. The graph of $y^2 = 8(x+3)$ and $y = 2x+k$ may have 2, 1, or no points in common. Determine k, such that (a) there are two points, (b) one point, or (c) no points of intersection; (d) draw the graph of the parabola and the line for each of the three cases.
12. Using acquired skills with ease, speed, and accuracy; developing the habit of consulting textbooks, if more explanation or practice is needed.	12. In each case, read the question carefully and give your answer in simplest form: $3^2 =\ ;\ -3^2 =\ ;\ (-3)^2 =$ $2^3 =\ ;\ -2^3 =\ ;\ (-2)^3 =$ $\sqrt{9} =\ ;\ \sqrt{a^2} =\ ;\ \sqrt{16x^3y^5} =\ ;$ $\sqrt{5^2-4^2} =\ ;\ \sqrt{5^2+4^2} =\ ;$ $\sqrt{18} + \sqrt{32} =\ ;\ \sqrt{18}\ \sqrt{32} =\ ;$ $\sqrt[3]{16} + \sqrt[3]{54} =\ ;\ \sqrt[3]{64x^7y^{-2}} =\ ;$ $x^{\frac{1}{3}} \cdot x^{\frac{4}{5}} =\ ;\ \left(a^{\frac{1}{2}}\ b^{\frac{2}{3}}\right)^6 =\ ;$

	$\left(\frac{7}{11}x^{-\frac{1}{2}}y^{\frac{11}{3}}\right)^0 =$; $5x^0 =$; $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right) =$; $\left(\sqrt{25}+3\right)\left(\sqrt{25}-3\right) =$; The linear factors of (x^4-1), over C, are ___; the real factors of (x^6-1) are ___; the sum of the roots of $3x^2-x+5=0$ is ____; the equation with integral coefficients and degree 3 has roots 5 and $2-\sqrt{3}$; the third root is ____; the fourth degree equation with real coefficients has one root $\frac{5}{2}-i\sqrt{3}$; it must have a root _____.
B. RELATIONS AND FUNCTIONS	
1. Distinguishing between relations that are or are not functions, defined by sets, equations, or illustrated by graphs.	1. Which of the following relations are functions? (a) $\{(x,y): y = 2x+3\}$ (b) $\{(x,y): \lvert x\rvert < 2\}$ (c) $\{(1,-2),(2,-1),(3,0),(2,1),(1,2)\}$ (d) $\{(x,y): y = 2\sqrt{x-2}\}$.
2. "Reading" into a defining equation: - x-intercept(s) - y-intercept(s) - symmetry - domain, range, and extent Applying all these "readings" to graphing.	2. (a) Give the coordinates of the x-intercept(s), y-intercept(s), domain, range, extent, and symmetry. Draw the graph: $y = x^5-x^3$; $y = x^4-8x^2+16$. (b) The graph of $y = x^3$ is symmetric about (0,0); can you find any symmetry for $y = x^3 - 8$? If so, give the coordinates of points symmetric to (2,0); to (-1,-9).

3. Understanding how to restrict domain, range, or both to change a relation that is not a function to one that is a function.	3. (a) Restrict the range so that $x^2 + y^2 = 25$ defines a function. (b) Are $y^2 = x^2-4$ and $y = \sqrt{x^2-4}$ the same relations? Draw the graph of each and, if you think that there is a difference, explain.
4. Using the different definitions of "$\sqrt{\ }$" carefully: $\sqrt{a}$ is always non-negative; $\sqrt{x^2} = \|x\|$.	4. If $y = \sqrt{5-x^2}$ is to be equal to $y^2 = 5-x^2$, what restriction must be added? Give domain and range for $y = \sqrt{5-x^2}$, and $y^2 = 5-x^2$.
5. Recognizing the effect on the graph of a relation when $f(x)$ is replaced by $f(x+h)$; $f(x) + k$; $f(x+h) + k$; $cf(x)$, $f(cx)$, $\|f(x)\|$, $f(\|x\|)$.	5. Given the function graph of g and non-zero real numbers h, k, and c, draw the graphs listed below: (a) $g(x) + 2$ (b) $g(-x)$ (c) $g(x+1)+2$ (d) $\|g(x)\|$ (e) $g(\|x\|)$ 

6. Drawing graphs of functions that are sums, differences, products, and reciprocals of functions, i.e., $f \pm g$; $f \cdot g$; $\frac{1}{f(x)}$. Recognizing the importance of vertical and horizontal asymptotes.	6. (a) $f: x \rightarrow \|x\|-2$, $g: x \rightarrow x$. In the same set of axes draw the graphs of f, g, and $f+g$. (b) Draw the graph of $y = \frac{1}{x^2-2}$ from the graph of $y = x^2-2$. Give the equations of the asymptotes.

C. COMPOSITION AND INVERSES

1. Finding the composite of functions, stating domain and range, drawing their graphs, recognizing the relation of corresponding points $(a,b) \rightarrow (b,a)$. Using notation $f \circ g$ or fg (distinct from product $f \cdot g$).	1. Given $f(x) = 2x-3$, $g(x) = x^2+1$; (a) Write expressions for: $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$. (b) Given: $f(x) = 2x+3$, $g(x) = \frac{1}{2}(x-3)$, express and simplify: fg and gf. (c) Given: $f(x) = \frac{1}{x}$; $g(x) = x^2-4$; give the domain of each function; find fg and gf and give domain and range of each. (d) Repeat for $f(x) = x^2$, $g(x) = \sqrt{x}$. (e) Show that $y = 5-3x$ and $y = \frac{5}{3} - \frac{1}{3}x$ are inverses. (f) $y = 4x^2-3$ and $x \geq 0$. Write the equation for f^{-1} and draw the graph of f and f^{-1}. (g) Find f^{-1} for $y = ax+b$, $a \neq 0$; $y = \sqrt{x^2+3}$, $x \geq 0$; $y = \sqrt[3]{1+x^5}$.

(h) If $f(x) = \frac{4}{x-1}$, $x > 1$ and

$g(x) = 2x$,
find the solution set of:
$gf^{-1}(x) = g^{-1}(f(x))$.

(i) Given the function $f(x) =$

$1 - \frac{1}{2}x$, defined on $[-4,4]$,

(1) Give the range of f.
(2) Write an equation for the

function g, if $g(x) = \frac{1}{f(x)}$:

give the domain and range of g.
(3) In the same set of axes, draw the graphs of f and g.
(4) Write the equation for f^{-1}, and, in the same set of axes, draw the graph of f and f^{-1}.

D. TRIGONOMETRIC AND CIRCULAR FUNCTIONS

1. Using degree and radian measures, relating them to revolutions and converting from one measure to another.
Understanding that this conversion allows us to treat the trigonometric functions of any angle as equal to the circular functions and vice versa.

1. (a) Complete the following table:

Revolution	Degree	Radian
1		
	1	
		1
$\frac{1}{4}$		
		$\frac{\pi}{6}$
	120	
$\frac{2}{5}$		
		$\frac{3\pi}{2}$

(b) Since $(45^\circ, \tan 45^\circ) =$

$(\frac{\pi}{4}, \tan\frac{\pi}{4})$, $\frac{\pi}{5}$, $\sin\frac{\pi}{5} =$

(,) ;

$(15^\circ, \cos 15^\circ) =$ ( ) ;
and generally, $(a^\circ, f(a^\circ)) =$

$(\frac{a}{180}\pi,$).

2. Applying the formulas for arc length and area of a circular sector, in terms of radius and central angle, with r=1, the special, important case.	2. (a) A hexagon ABCDEF is inscribed in circle O with radius 1. Arc AR measures $\frac{3}{2}$ radians. Is point R closer to point B or point C? (b) The area of a circular sector with central angle 0.4 radians is 20 square units. Find the radius of the circle and the arc length of the sector. (c) A phonograph record, 12 cm in diameter, turns at $33 \frac{1}{3}$ RPM (revolutions per minute). A point on the rim of the record travels through how many centimeters, while a 3-minute song is being played?
3. Recognizing and working with Cartesian (rectangular) as well as polar coordinates of points and converting from one to the other.	3. Give the polar coordinates for each of the following points: $(-3,3)$; $(4,-4\sqrt{3})$, $(2\sqrt{3},-2)$; give the rectangular coordinates of each of the following points: $(2, \frac{\pi}{6})$; $(2, -\frac{11\pi}{6})$; $(2,225)^\circ$.
4. Using the concepts of angle and unit circle in standard position; positive, negative, coterminal and reference angles, and periodicity.	4. (a) Give the reference angle of each of the following: 113° ; 3 radians ; -250° ; 755° . (b) Replace the angle by the reference angle and complete: $\sin(\frac{11\pi}{6}) =$; $\tan(-115^\circ) =$; $\sec(\frac{7\pi}{2}) =$; $\csc(4\pi) =$. If θ is in quadrant III, $\cos(\theta+2n\pi) =$; $\tan(\theta+5\pi) =$.

5. Using the definitions of the six trigonometric functions; making use of unit circle and 30-60-90, and 45-45-90 triangles to evaluate special functions; understanding that if $f(\theta)=s$, s is unique for a given angle θ, but each function value is assumed by infinitely many angles.

5. Give all angles in degrees and radians,
   (a) for which $\cos \theta = \frac{1}{2}$.

   (b) Complete the following table for angle θ in standard position, P on terminal side of angle and on unit circle.

Degrees	Radius	Exact Values of			Coordinates of P	
$0<\theta<360°$	$0<\theta<2\pi$	Sin θ	Cos θ	Tan θ	Polar	Cartesian
45						
		$\frac{1}{2}$				
				$\sqrt{3}$		
	$\frac{7\pi}{4}$					
			-1			
						$\left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$
					$\left(1, \frac{3}{4}\pi\right)$	

6. Drawing the graphs of the six circular (trigonometric) functions over a given domain, recognizing period, range, even or odd function; asymptotes for the tangent and the reciprocal functions.

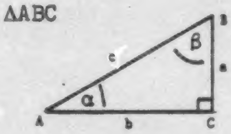
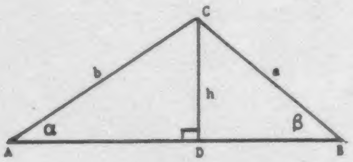
6. (a) The graph of $y=x^2=(-x)^2$ is symmetric about the y-axis; it is an even function. Which of the trigonometric functions are even?

   (b) Give the coordinates of the point on the graph of y = sin x, symmetric to

   $(\frac{5\pi}{6}, \frac{1}{2})$ and closest to the

   y-axis: (,   ) ;
   sin(-x) = ______, which illustrates that the sine function is _______, that it is symmetric about _______.

   (c) Draw, in the same set of axes, for $-\pi \leq \theta \leq \pi$, the graphs of $y = \tan \theta$ and $y = \cot \theta$.

7. Understanding and applying the concepts of amplitude, period, and phase shift and drawing graphs of $y = A\sin(Bx+C)$, $y = A\cos(Bx+C)$, and $y = A\tan(Bx+C)$. Recognizing, in graphing, what portion of the graph only needs to be plotted point by point, due to symmetry and periodicity.	7. Give the period, amplitude, and phase shift of $y = 3\sin(2x - \frac{\pi}{2})$ and draw the graph.
8. Applying the trigonometric functions to right triangles and solving problems that the Pythagorean theorem fails to explain.	8. Given right ΔABC as marked;  (a) Place angle A in standard position, with $\overline{AC}$ the initial side; (b) Repeat with angle B and $\overline{BC}$ the initial side. Using the diagrams, complete: $\sin\alpha =$; $\sin\beta =$; $\cos\alpha =$; $\cos\beta =$; $\sin^2\alpha + \cos^2\alpha =$; $\sin^2\beta + \cos^2\beta =$; it follows that $\sin\alpha =$ cos____, and $\cos\alpha =$_____; and $\alpha + \beta =$ ____. $\sin 45^\circ = \cos$_____?; $\cos 60^\circ = \sin$_____?; $\sin^2 60 + \cos^2 60 =$ ______=____. (c) The angle of depression of a raft from the top of a tower, 48 ft above water level, is 24°. How far is the raft from a point directly below the tower? (Use a calculator.) (d) Given ΔABC with altitude $h=\overline{CD}$;  express h in rt.ΔADC; in rt.ΔBDC; and prove: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$.

9. Applying and understanding, generally, the relationships of cofunctions and functions.

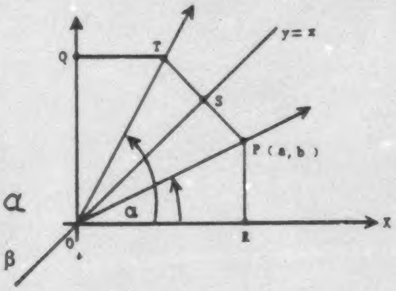
9. The terminal side of ∢ POX is reflected in the line y=x with point T the reflection of point P.

PR ⊥ $\overrightarrow{OX}$

TQ ⊥ $\overrightarrow{OY}$

m ∢ XOP = α

m ∢ XOT = β



In terms of a and b, give the coordinates of points R,T,S,Q; m ∢ to Q = ?; β = ?; Draw ⊥ from T to $\overrightarrow{OX}$;

Prove: $\sin\alpha = \cos(90 - \alpha)$,
$\cos\alpha = \sin(90 - \alpha)$,
$\cot\alpha = \tan(90 - \alpha)$.

10. Stating and applying the Law of Sines and the Law of Cosines; using the formula for the area of a triangle

$K = \frac{1}{2} ab \sin\gamma$.

10. (a) Given ΔABC, b=25, c=15, m ∢ B=60°;

(b) b=15, c=25, m ∢ B=60°; in each case find sin C.

(c) Find all parts of ΔABC, AC=7, BC=10, and m ∢ B=42°. Construct each triangle above and explain results.

(d) The sides of a triangle are 6, 12, and 14 units. Is the triangle acute or obtuse?

(e) Find the length of the side of a regular octagon ABCDEFGH, if diagonal AE = 20 cm.

11. Deriving from $\sin^2\theta + \cos^2\theta = 1$, the other Pythagorean Identities and applying them to prove identities and solve equations.

11. Using Pythagorean Identities and definitions of functions,

(a) simplify:

$\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta}$;

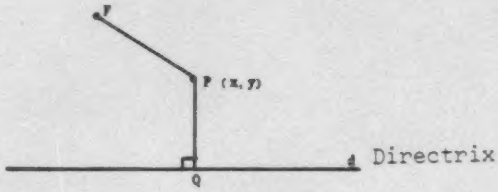
$\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}$;

$(\csc y-\cot y)(1+\cos y)$;

$\frac{\sec\theta-\cos\theta}{3\tan\theta\sin\theta}$.

	(b) Prove that each of the following is an identity: $\cos^4 x-\sin^4 x=1-2\sin^2 x$; $\frac{2\sin^2\theta-1}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$; $\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A} = 1 - \sin A\cos A$. (c) Solve for all x, $0 \leq x < 2\pi$ $3 \sin^2 x - \cos^2 x = 0$; $\sec^2 x + \sec x -1 = 0$; $\sec\theta + \cos\theta = \tan\theta \sin\theta - 1$.
12. Applying distance formula and law of cosines to derive identity for $\cos(\alpha -\beta)$; stating and applying all the Addition, Subtraction, Multiple, and Half-angle Identities for sine, cosine, and tangent functions. Applying to evaluate expressions, prove further identities, and solve equations.	12. (a) Simplify and, when possible, give exact value: $\sin 75^\circ$; $\cos 255^\circ$; $2\sin \frac{\pi}{5} \cos \frac{\pi}{5}$; $2 \cos^2(15^\circ)-1$; $1-2\sin^2 (\frac{3\pi}{8})$; $1 + \tan^2 (\frac{\pi}{6})$. (b) Use the addition identity for $\cos (\alpha - \beta)$ to prove that $\cos (- \alpha) = \cos (0 - \alpha) = \cos\alpha$, and (c) $\cos (\alpha + \beta) = \cos (\alpha - (-\beta)) = \cos\alpha \cos\beta - \sin \alpha \sin\beta$. Any fact missing? (d) Solve for $0 \leq x < 360^\circ$: $2 \cos 2x + \sin 2x \sec x = 0$; $\cos 4x = 1-3 \cos 2x$. (Hint: $\cos 4x = 2 \cos^2 2x-1$.)
13. Evaluating expressions involving inverse circular functions; taking note of symbols used, Sin x, Cos x, Tan x, and $\text{Sin}^{-1} a$ or Arc sin a, $\text{Cos}^{-1} a$, . . . etc; drawing graphs of inverse functions.	13. (a) Evaluate: $\text{Sin}^{-1}(-1)$; $\text{Tan}^{-1} \left(-\frac{\sqrt{3}}{8}\right)$; $\text{Cos}^{-1} \left(-\frac{\sqrt{3}}{2}\right)$; $\cos (\text{Sin}^{-1} (-\frac{1}{2}) + \text{Cos}^{-1} (-\frac{1}{2}))$; $\tan (\text{Sin}^{-1} (\frac{1}{2}) + \text{Cos}^{-1} (\frac{1}{2}))$. (b) Show that $\text{Tan}^{-1}(\frac{1}{2}) + \text{Tan}^{-1}(\frac{1}{3}) = \frac{\pi}{4}$.

	(c) Draw the graph of Sin x and reflect in the line $y = x$. Give the coordinates of the points corresponding to $(-\frac{\pi}{2}, -1)$, and $(\frac{\pi}{2}, 1)$. What function defines this graph? (d) Solve for x: $\text{Cos}^{-1}(2x) = \text{Sin}^{-1}(x)$.
14. Following the derivation of formulas $\sin A \pm \sin B$ and $\cos A \pm \cos B$ and applying in problems.	14. (a) Write the addition formulas for $\sin(\alpha + \beta)$ and $\sin(\alpha - \beta)$ and add; let $\alpha + \beta = A$ and $\alpha - \beta = B$ and check your results with $\sin A + \sin B =$ $2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$. Repeat for $\cos A \pm \cos B$. (b) Write as a product $\sin\frac{\pi}{3} + \sin\frac{\pi}{4}$; write as a product and evaluate: $\sin \frac{17\pi}{12} - \sin \frac{11\pi}{12}$. (c) Prove the identity: $\frac{\cos 7t + \cos 5t}{\sin 7t - \sin 5t} = \frac{\csc t}{\sec t}$.
E. CONIC SECTIONS 1. Drawing a sketch showing the planes cutting a right circular cone and the resulting curve. Defining the circle, parabola, ellipse, and hyperbola as a set of points. Defining the curves in terms of 2 distances and their ratio. Recognizing the conic section by the value of e, $0 < e \leq 1$, or $e > 1$.	1. (a) What will be the curve when the cutting plane is parallel to an element (edge) of the cone? Under what condition will the curve be a hyperbola? Two straight lines? (b) State the definition of an ellipse, using 2 fixed points and a constant distance.

	(c) Use points F,P, and Q and the eccentricity 0 < e, and define all conic sections. 
2. Using with ease and accuracy the formulas for distance, midpoint, and distance of a point from a line, $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$.	2. Given: P(x,y), and C(-2,3): (a) Write an expression for the length of $\overline{PC}$. (b) State the fact that PC=4. (c) What is the graph of the set of points, (x,y), described in (b) above? (d) Use the definition of a circle and give its equation if a diameter $\overline{AB}$ has end points at A(-3,-4), and B(3,4). (e) Write the equation of the circle with center at (2,3), and tangent to the line $y = \frac{3}{4}x - 1$.
3. Recognizing the equation of an ellipse. Identifying and computing major axis, minor axis, equations of directrices, length of latus rectum, value of e. Stating coordinates of vertices, foci.	3. (a) Find the length of the major and minor axis, the coordinates of the foci, the end points of the latera recta, and the equations of the directrices, of $16x^2 + 25y^2 = 400$. (b) Give the equations of the circles tangent to the ellipse above at the x-intercepts; at the y-intercepts.

ADVANCED MATHEMATICS E

4. Recognizing the equation of a hyperbola; stating its domain; identifying transverse and conjugate axes, vertices, foci, eccentricity, equations of asymptotes.	4. Write the equation of the hyperbola (a) with vertices at $(\pm 4,0)$, and foci at $(\pm 5,0)$; (b) with vertices at $(0,\pm 4)$, and asymptotes $y = \pm \frac{4}{3}x$.
5. Recognizing the equation of a parabola and its axis of symmetry. Identifying vertex, focus, length of latus rectum, equation of directrix.	5. Find the equation of the parabola: with vertex at $(0,0)$, $F(3,0)$; vertex at $(0,0)$, $F(-3,0)$; give the equation of the directrix in each case.
6. Analyzing coefficients of general quadratic, including degenerate cases.	6. Given $Ax^2+By^2+Cx+Dy+E = 0$, A,B,C,D,E real numbers, under what conditions will the graph be: (a) a circle (b) an ellipse (c) a hyperbola (d) a parabola (e) a straight line (f) two straight lines
7. Rewriting the equation of a conic in standard form, identifying the curve, stating its characteristics, sketching the graph, when the center (vertex) is translated from (0,0), to (h,k).	7. (a) Give the standard form of each of the following; give the coordinates of the center, vertices, (focus,vertex), foci, equation(s) of directrix; and sketch the curve: $x^2+y^2-4x+6y-3 = 0$; $y^2-8x-2y-7 = 0$; $x^2+4y^2-2x+8y-21 = 0$. (b) The circle defined by $x^2+(y-3)^2=r^2$ will have four points in common with the curve $\frac{y^2}{25} - \frac{x^2}{9} = 1$, if r is ?
8. Solving systems of equations of conics algebraically.	8. (a) Solve the systems algebraically: $\begin{cases} y^2-8x+16 = 0 \\ x^2+y^2-4x-16 = 0 \end{cases}$

	(b) Write each component equation in standard form and solve graphically. (c) Repeat (a) for: $\begin{cases} x^2-y^2 = 1 \\ x^2+y^2 = 7 \end{cases}$ and check by solving graphically.
F. GRAPHS OF POLAR EQUATIONS AND PARAMETRIC EQUATIONS	
1. Plotting points in the real number plane, given Cartesian or polar coordinates.	1. Give the polar coordinates for: $(1,1)$; $(-1,-\sqrt{3})$; $(-2,2\sqrt{3})$; Give the rectangular coordinates for: $(3, -\frac{2\pi}{3})$; $(5, \frac{\pi}{2})$; $(2, \frac{3\pi}{2})$.
2. Using the relations $x=r\cos\theta$ and $y=r\sin\theta$ to translate equations from one system to the other.	2. (a) Write the polar equations for: $x = -2$; $y = 5$; $y = x$; $x^2+y^2+2y = 0$. (b) Write the Cartesian equations for: $r = 4$; $\theta = \frac{\pi}{3}$ $r = 2\sin\theta + 4\cos\theta$. (c) $y = 5$ means $0x+y=5$; any point with ordinate 5 will be on the graph of the line; compare with equations: $r = 4$, $\theta = \frac{\pi}{3}$, and explain. (d) Are the graphs of $r^2+2r\sin\theta = 0$ and $r+2\sin\theta = 0$ the same? Explain your answer.

3. Recognizing the polar equations for the straight line, circle, cardioid; making a table of values, drawing graphs, and finding points of intersection.

3. (a) Draw the graph of: $r = \sin\theta - 1$; $r = 2 \sin\theta$.
   (b) Draw the graphs of: $r = 2+\cos\theta$ and $r = 5 \cos\theta$ and give all points, (r,θ), of intersection.
   (c) Draw the graph of $r = \sin 2\theta$. First complete the table.

θ	0°.	30°	45	60	. . . . .	360
$r = \sin 2\theta$	0	$\frac{1}{2}\sqrt{3}$	1			

4. Drawing the graph of parametric equations; translating them into Cartesian equations.

4. Make a table of values
   (a) and sketch the graph of

   $x=\frac{1}{2}t$, $y=3-t^2$;

   (b) eliminate t and write the Cartesian equation.

   (c) $x = 4 \sec \theta$ and $y = 3 \tan \theta$ are the parametric equations of a hyperbola. Give the Cartesian equation and draw the graph.

   (d) Draw the graph of $x=t^2-1$, $y=t-1$ and give the Cartesian equation of the curve.

G. LIMITS OF FUNCTIONS

1. Relating the limit of a sequence to the "associated" function.

1. Write the terms of $\{\frac{1}{3}n + 2\}_1^\infty$
   (a) and draw the graph in a Cartesian plane.

   (b) In the same plane draw the graph of $y = \frac{1}{3}x + 2$

ADVANCED MATHEMATICS G

2. Applying the concept of interval and neighborhood to functions; drawing $\xi - \delta$ regions and writing absolute value statements.	2. Draw the graph of: $\left\{\frac{2+k}{k}\right\}_1^n$ and $f: x \to \frac{2+x}{x}$ in the same coordinate axes. (a) Find M such that $a_n < 2$ for $n \geq M$. (b) Find M such that, for $n \geq M$, $\|a_n - 1\| < 0.1$. (c) Find N, such that for $x > N$ $\left\|\frac{2+x}{x} - 1\right\| < 0.1$.
3. Recognizing the behavior of a function as $x \to x_0$, $x \to \pm\infty$, from the function and its graph.	3. (a) Are the functions $f(x) = \frac{x^2-9}{x-3}$ and $g(x)=x+3$ equivalent? (b) Draw the graph of each. (c) $\lim\limits_{x\to 3} \frac{x^2-9}{x-3} =$ (d) Find $\lim\limits_{x\to 0} (\frac{2+x}{x})$ and compare your answer with 2(c) above. (e) Find $\lim\limits_{x\to\infty} (\frac{2+x}{x})$
4. Using, without proof, the limit theorems, the concepts of left-hand and right-hand limits, and drawing the correct conclusion about existence of limit at that point.	4. Complete each of the following: (a) $\lim\limits_{x\to 2^-} \frac{x^2-4}{x-2} =$ (b) $\lim\limits_{x\to 2^+} \frac{x^2-4}{x-2} =$

	(c) Draw the graph of $f(x) =$ $\begin{cases} \frac{x^2-4}{x-2} & \text{for } x \neq 2 \\ 4 & \text{for } x = 2 \end{cases}$
5. Understanding the notion of the limiting position of a secant to a curve and the definition of the slope of a curve at a point.	5. Given the function f, such that $f(x) = \frac{1}{2}x^3 - 2$. (a) Find $f(-2)$, $f(-1)$, $f(0)$, $f(a)$, $f(2)$, $f(2+h)$. (b) Given points $A(-2, f(-2))$ and $B(2, f(2))$, write the equation of line AB. (c) Draw the graph of f and line AB. (d) Point A→point B, what happens to the line? (e) P_1 and P_2 are on the graph of f. $P_1(x_1,y_1)$, $P_2(x_1+h, y_1+h)$ Find the slope of $\overleftrightarrow{P_1P_2}$.
6. Writing the slope of a line through two points on a curve; finding the limit of the slope as the points approach each other and coincide.	6. Given the function $y = x^2 - x$: (a) Find the slopes m_1, m_2, m_3, ... of $\overleftrightarrow{AB_i}$ if $A(1,0)$ and $B_1(2,2)$; $B_2(1.5,y_2)$, $B_3(1.3,y_3)$, $B_4(1.1,y_4)$; $B_L((a+h), f(a+h))$; (b) Find the value of and simplify $\frac{f(a+h) - f(a)}{(a+h)-a}$; find limit of the above expression as $h \to 0$.

7. Using the definition of the derivative and applying it to polynomial functions, as well as $f'(x) = nx^{n-1}$.	7. (a) Use the definition of the derivative and check your result by direct differentiation: (1) $f(x) = 3x+4$; $g(x) = 3x-7$; (2) $f(x) = 4x^2-3x+5$; $g(x) = 4x^2-3x-2$ (3) $f(x) = \frac{1}{2}x^3+2$; $g(x)= \frac{1}{2}x^3-3$. (b) Evaluate each derivative above for $x = -2,0,1,3$. (c) (1) Find the derivative of $f: x \to x^3-12x+1$; (2) Evaluate $f'(x)$ at $x=-3,-1,0,1,3$. (3) Find the values of x for which the derivative is zero.
8. Using prior learning: symmetry of the graph about the y-axis if $f(x) = f(-x)$; about the origin if $f(-x) = -f(x)$, and combining all the facts with derivative and graphing, including horizontal tangent at x_0, if $f'(x_0) = 0$; applying to problems; taking care of restrictions of domain.	8. (a) Given: $f: x \to x^4-2x^2+5$ (1) Find $f'(x)$ (2) Find all values of x where $f'(x) = 0$; this implies____. (3) Evaluate $f(x)$ for $x = \pm3, \pm2, \pm1, 0$. (4) Using the above results draw the graph of f: (b) Use the quadratic function and its graph to find the dimensions of the largest area of a rectangle with perimeter 40 cm. Repeat the above, using the derivative. Compare results and discuss the methods used.
H. POLYNOMIAL FUNCTIONS	
1. Recognizing when an algebraic expression is a polynomial or a polynomial function, its degree and leading coefficient. Distinguishing between the zero polynomial and the constant function.	1. Which of the following are polynomials? Give the reason if some expressions are excluded. (a) $2x^5-4x^3+\frac{1}{2}x-\sqrt{3}$; (b) $f(x) = 7x + 5x^3 - \frac{2}{x^2} + 2$; (c) $y = 5$. (d) Give the degree of the polynomials above.

ADVANCED MATHEMATICS H

2. Evaluating polynomials using good judgment in choosing direct or synthetic substitution.	2. Evaluate for x = $\pm \frac{1}{2}$, ± 1, ± 2: (a) $2x^3-11x^2+15x-6$ (b) x^5-16
3. Listing the set of possible rational zeros of a polynomial and checking to find the zeros; checking, by inspection, whether or not 0, ∓ 1 are zeros.	3. List the possible rational zeros of (a) x^4-3x^2+5x+7 (b) $3x^4+4x^3-3x+2$ (c) x^5+1
4. Using Descartes' Rule of signs, locating real zeros between consecutive integers, and finding lower and upper bound of zeros.	4. $f(x) = x^4-3x^3-2x^2+3x-5$ (a) Find the number of positive zeros. (b) Find the number of negative zeros. (c) Find the lower and upper bound of zeros. (d) Locate, if Descartes' rule of signs applies, any real zeros between consecutive integers.
5. Using the factor and remainder theorem in problems and locating multiple zeros.	5. Factor the polynomial (a) $3x^3-8x^2+3x+2$ (b) Show that -2 is a double zero of $4x^5+4x^4-23x^3-39x^2-72x-10$ and write the polynomial as a product of linear factors over the complex numbers.
6. Applying all previous steps in this section to draw graphs of functions and solve problems.	6. Given: $f(x)=x^4-x^3-3x^2+x+2$ (a) Obtain all the information that will help you draw the graph of the function: zeros, x-intercepts, y-intercept, if possible $f'(x)$ and the zeros of $f'(x)$.

(b) A right triangle with hypotenuse of length 5 is rotated about one of its legs. Find the radius, height, and volume of the largest cone that can be generated.

I. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1. Using the law of exponents; simplifying and evaluating expressions; solving equations.

1. Simplify each of the following:

(a) $27^{\frac{1}{3}}$; $32^{\frac{1}{5}}$; $8^{-\frac{1}{3}}$; $(a^2b^3)^2$;

$(5^2+12^2)^{\frac{1}{2}}$; $\dfrac{3x^{-1}-x^{-2}}{x^{-3}}$.

(b) Find the value of x:
$3^{x-2}=27$; $(8x)^{-1}=64$.

2. Understanding the laws of logarithms; understanding and applying concepts of base, characteristic, mantissa; simplifying and evaluating expressions; solving equations.

2. (a) In the base 10, if log 2 = a, log 3 = b, and log 5 = c, complete the table:

N	8	8000		$\frac{54}{25}$	
log N			a+b+2		4a+b-3c

(b) Write in exponential form:
$\log_5 32 = x$; $\log_x 125 = 3$;

$\log_3 x = -4$.

(c) Solve for x:
$\log x + \log (x-3) = 1$;
$\log (x-2) < 0$.

In the above, would a change of the base make a difference? Explain.

(d) In each case, give the characteristic for the given number and base:
Number 4500 72 675
Base 10 ——; 2 ——; 5 ——;

(e) $2^{10}=1024\approx10^3 \Longrightarrow 2\approx10^{.3} \Longrightarrow$
$\log 2\approx0.30$
Compare this result with the entry in the log tables; try 7^2!

3. Extending powers from rational to irrational exponents: drawing graphs of $y = a^x$, $a > 0$, $a \neq 1$ $x \varepsilon$ Reals; understanding special bases, 10, and $e \approx 2.71728...$	3. Given $1.41 < \sqrt{2} < 1.42$: (a) What can you say about the order of: $2^{1.41}$, $2^{\sqrt{2}}$, $2^{1.42}$? (b) Plot points of $f:x\rightarrow 2^x, x=-5,-4,-3,\ldots,5,\ldots$ and draw a smooth line connecting them. What is the extent of the graph? Check the value of $2^{\sqrt{2}}$? Check with calculator reading. (c) Repeat for $g:x\rightarrow 3^x$. (d) If the graphs of 2^x and 3^x were drawn in the same set of axes, where would the graph of $y=e^x$, $e\approx2.72$ be, in relation to the other graphs? (e) Whether the base of $y=a^x$ is 2,3,e, or any other real number $a\neq1$ and $a > 0$, the range of these functions is: the set . . . ?
4. Recognizing the logarithm function as the inverse of the strictly increasing (one-to-one) function, the exponential function.	4. Draw the graph of $y=2^x$ and reflect it into the line: (a) $y = x$. (b) Write the equation of this graph. (c) Give domains of 2^x and its inverse; give the ranges of both functions. (d) What is the extent of the inverse? (e) If $f:x\rightarrow a^x$, and $g:x\rightarrow \log_a x$, the point (0,1) corresponds to the point (,)? Is this true for all a in the domain of a?
5. Drawing the graph of a^x, a^{-x}, $-a^x$; comparing for different bases.	5. (a) How can you obtain the graph of 2^{-x} from the graph of 2^x? (b) In which quadrant(s) is the graph of $y=-e^{-x}$?

6. Recognizing that

$\log_b b^x = b^{\log_b x} = x$

and applying in evaluations.

6. Evaluate:

(a) $2^{\log_2 8}$; $2^{\log_2 32}$; $5^{\log_5 125}$;

$10^{\log_{10} 1000}$.

(b) if $f(x) = 2^x$, $f^{-1}(x) =$;
if $f(x) = \log_2(x)$, $f^{-1}(x) =$.

$f^{-1}(f(x)) = \log_5(5^x)$ and

$f(x) = 5^x$, then $f^{-1}(x) =$.

(c) The number e, the base of the natural logarithms, and the exponential function e^x, is defined as:

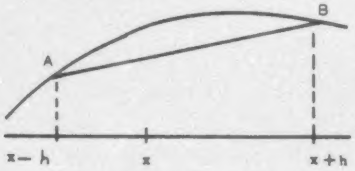
$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots,$$

Approximate e by adding the first 7 terms. (Note, if you have no calculator, that, $3! = 3(2!)$; that $= \frac{1}{4!} = \frac{1}{4}\left(\frac{1}{3!}\right)$, etc.)

7. Applying exponential functions to problem solving.

7. An amount of $1000 is invested at 6% for n years, compounded

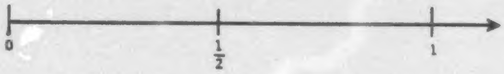
(a) semiannually;
(b) continuously;
write, in simplified form, the equations for (a) and (b);
(c) For (a) above, after how many years will the investment double? Simplify equation, and use, if available, natural log table or calculator.
(d) The count of bacteria in a culture after 4 days was 202,500, where it was only 90,000 after 2 days. What was the initial count?
(e) The half-life of a radioactive substance is 25 minutes. What fraction of the sample will remain after 12.5 min.; 75 min.; 150 min.?

8. Using log tables; understanding interpolation, but developing the ability to compute effectively, using a calculator; or, writing a program, depending on the problem and on one's preparation in a separate computer course.	8. (a) The graph of a^x is concave If you join two points on the graph, does the segment lie below or above the graph? (b) Make a corresponding statement about the function $y = \log_a x$. (c) If you use a value between 2 readings on the graph, at x, and approximate the function by the line segment, will you overestimate or underestimate? What does this mean for interpolation in the case of logarithms? 

J. SEQUENCES - SERIES - LIMITS

1. Recognizing that sequences and series are functions with the natural numbers for domain.	1. Write the given mapping using (a) subscripts; (b) function notation. $\begin{array}{ccc} 1 & 2 & 3\ \ldots \\ \downarrow & \downarrow & \downarrow \\ -3 & 0 & 3 \end{array}$
2. Discovering a pattern, given the first three terms.	2. Give the next three terms of the sequence 0.01, 0.0101, 0.01001,...
3. Writing the terms of a sequence or series, given a formula for the general term, or a recursive definition.	3. Write the first six terms if $a_1=3$, and $a_{n+1}=(-1)^n a_n$.
4. Using, with facility, the symbols $\{a_n\}_1^n$, $\sum_{k=1}^{n} a_k$, $\sum_{i=1}^{\infty} a_i$	4. Write each term of (a) $\{2^i\}_1^5$; (b) $\sum_{i=1}^{5} ix^{2i}$.

ADVANCED MATHEMATICS J

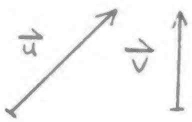
5. Drawing the graph of a sequence; recognizing from the graph and applying in problems the concepts of limit; upper bound, lower bound, (lub, glb); interval and neighborhood in which all but a finite number of terms are: nonincreasing, nondecreasing.	5. Draw the graph of $\{2^{-k}\}_1^n$ (a) on the number line.  (1) $\lim_{k\to\infty} 2^{-k}$ = ? (2) a_{10} = ? (3) For what K is $\lvert a_k - L\rvert < \frac{1}{2048}$? (4) Give the open interval in which all a_i, $i \geq 10$ are found. (b) In the plane: (1) Connect the points by a smooth line. (2) How does the graph of the sequence differ from that of the line? (3) Can you give the equation of the curve of (1)? (4) State the range of $\{2^{-k}\}_1^{100}$ and of $f(x)=2^{-x}$ on the same interval.
6. Recognizing and applying in problems arithmetic and geometric series, finite and infinite.	6. (a) Find the sum of all 3-digit numbers divisible by 6. (b) Find the sum of $\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots + \frac{9}{10^n}$. (c) Find the limit of the sum in (b) above, as $n\to\infty$.
7. Using the concept of inserting arithmetic means, handling the geometric mean algebraically and geometrically.	7. (a) Insert four arithmetic means between 9 and 24. (b) Given segments of length a and b, construct segment x of length $\sqrt{ab}$.
8. Recognizing convergence and divergence of sequences and series, and finding the limit if it exists.	8. (a) Find the $\lim_{n\to\infty} \frac{(n-1)(n+1)(3n-2)}{5n^3}$ (b) Determine x if the infinite series $x^2-x^3+x^4-\ldots$ converges to $\frac{x}{5}$.

9. Applying the formulas for sums to problems.	9. (a) The midpoints of the sides of an equilateral triangle, with side 12 units, are joined, forming another triangle. This process is continued indefinitely. Find the sum of all perimeters and of all areas formed. (b) Give the rational number equivalent to $1.2\overline{34}$, using an infinite geometric series.
K. PERMUTATIONS AND COMBINATIONS	
1. Understanding and applying factorial notation.	1. (a) If $8! = 40{,}320$, $10! = ?$ (b) Simplify $\frac{n!r!}{(n-1)!(r+1)!}$.
2. Understanding and applying the counting principles: (a) multiplication (b) addition	2. (a) In how many ways can you jog home from school via a store if there are two roads from school to the store and three roads from the store to home? (b) Given digits 0,1,3,4,5, how many numbers greater than 2000 can you form, using each digit once only? (Hint: how many four-, how many five-digit numbers?)
3. Distinguishing between permutations and combinations and using and interpreting the different symbols: $P(n,r,)$, $_nP_r$; $C(n,r,)$, $_nC_r$, $\binom{n}{r}$.	3. (a) Find the value of n, given: $P(n,4) = 3(P(n,3))$; $C(n,15) = C(n,3)$. (b) How many different sums of money can you make if you have a \$1 bill, a \$5 bill, a \$10 bill, and a \$20 bill? (c) A coach has k different batting orders for his 9 baseball players. In terms of k, how many batting orders does he have if the pitcher bats last?

4. Solving problems involving permutations, permutations with repetitions, circular permutations. (Use a calculator when needed!)	4. (a) A nickel, a dime, and a quarter are tossed simultaneously. In how many different ways can the coins land? (b) In how many ways can 9 students in a classroom with 14 desks be seated? (c) How many "words" can be formed using all the letters of the word "statistics?" (d) A license plate has 6 symbols: 3 letters followed by 3 digits. How many different license plates can be made under the following conditions? (1) The letter O cannot be used and the digit 3 is at the end. (2) All letters must be different. (3) All letters must be different and the 0 (zero) digit is excluded. (e) Fifteen distinguishable pearls (beads) are arranged in a circle. In how many ways can this be done, if: (1) they are placed on a table top? (2) they are placed on a string with a clasp? (3) they are placed on a string without a clasp?
L. THE BINOMIAL THEOREM	
1. Discovering by observation and actual expansion of $(a+b)^0$, $(a+b)^1$, $(a+b)^2$, ... $(a+b)^5$, the pattern of the coefficients, the relation of the coefficients of $(a+b)^{n-1}$ to those of $(a+b)^n$.	1. Give the number of terms, the sum of the exponents of each term, and each coefficient in the form: $\binom{n}{k}_1$ of $(a+b)^5$.
2. Recalling and using Pascal's triangle.	2. In the expansion of $(a+b)^h$, the coefficicnts are 1 k 45 120...; find h, k, and the coefficients of the 4th and 5th terms of $(a+b)^{h+1}$.

3. Being able to write any term or the entire expansion of $(a+b)^n$, using binomial coefficients in the form $C(n,r)$ or $\binom{n}{r}$.	3. Give the 7th term of $(2x - \frac{1}{4}y)^{15}$.
4. Applying the binomial theorem in problems.	4. Use the binomial theorem to find the value of $10,000 after 10 years at the low interest rate of 2% compounded annually. How many terms are needed to compute $(1+.02)^{10}$ to 4 decimal places? Compare your answer with the calculator reading.
M. MATHEMATICAL INDUCTION	
1. Appreciating the power of this new method of proof and the reason for introducing it.	1. Can you use the types of proofs from algebra and geometry to establish that the sum of 1+2+3+...+n exists and tell whether or not its value for n is very large?
2. Stating the Principle of Mathematical Induction.	2. What is the relation of the natural numbers to the sequence of statements $\{P_i\}_1^n$, to be proved?
3. Saving time by going beyond proving that P(1) holds; remembering the importance of counter examples.	3. Based on $1^2+1+41 = 43$ $2^2+2+41 = 47$ $3^2+3+41 = 53, \ldots$ prove or disprove that for any $n \in N$, n^2+n+41 is prime.
4. Developing the ability to guess the solution of a formula that observes a pattern.	4. 1+3 = 4; 1+3+5 = 9; 1+3+5+7 = 16; (a) $1+3+5+\ldots+(2n-1) = ?$ (b) 1=1 1+2=3 1+2+3+4+...+9+10+11+12 = ? 6 pairs of numbers, sum = ?

5. Using mathematical induction to prove statements where direct and indirect proofs fail.	5. Prove by mathematical induction: (a) Your guesses in 4(a) and (b). (b) Remember the "tricks" used to obtain the formulas for the sums of arithmetic and geometric series. Prove them. (c) Prove that for all $n \in N$, $\cos(\theta+n\pi)=(-1)^n \cos\theta$. (d) Prove that for every $n \in N$, n^3+2n has a factor of 3.
N. PROBABILITY 1. Understanding the concept of sample space and event; drawing a diagram and computing simple probabilities.	1. (a) Draw a tree diagram to find the probability of obtaining at least 2 heads when tossing 3 coins (1 coin 3 times). (b) Two different numbers are chosen from the numbers 1,2,3,4. What is the probability that the sum is greater than 4? (c) Thirteen cards of a 52-card deck are dealt. What is the probability that all 13 cards are red? (Simplify and use a calculator.)
2. Being able to use a Venn diagram and explaining $P(A \cap B)$, $P(A \cup B)$; a reduced sample space for $P(B/A)$.	2. (a) Let E be the event that a face card (J,Q,K) or a heart will appear on a draw of 1 card from a deck of cards. Find P(E). (b) Find the probability of getting a sum of 8, 9, or 10 rolling two dice, given that the first die shows a six.
3. Solving problems by applying the binomial probability function.	3. (a) A baseball player's batting average is .250. What is the probability of getting exactly 1 hit in the next 4 times at bat?

	(b) What is the probability of getting at least 7 out of 10 answers correct on a true-false test by guessing correctly with probability $\frac{2}{3}$?
4. Being able to compute the expected value of a random experiment.	4. (a) On a roll of one die, an odd number pays 50 cents, a six pays 60 cents, and a two pays 90 cents. Find the expected value. (b) A father offers to pay his son $90 if he earns all A's, or $10 for each A he makes. The boy is a good student, and estimates his probabilities of earning A's to be: Algebra . . . 0.9 English . . . 0.7 Chemistry . . 0.8 Spanish . . . 0.6 (1) Calculate his mathematical expectation if he chooses $10 per A. (2) Calculate his probability of making all A's. (3) Calculate his mathematical expectation if he chooses $90 for earning all A's. (4) Which offer should he choose?
O. VECTORS 1. Drawing vectors that are equal to a given vector; representing the sum, difference, scalar multiples, or linear combinations of two or more given vectors.	1. Given vectors $\vec{u}$, and $\vec{v}$,  draw: (a) $\vec{u}+2\vec{v}$; (b) $2\vec{u}+\vec{v}$; (c) $\vec{r}=\vec{v}$; (d) $(\vec{u})(\vec{v})$; (e) $3\vec{u}-2\vec{v}$.

2. Applying operations to vectors on the basis of new definitions and the algebra of vectors.	2. (a) Show algebraically and graphically that, for scalars r and s, $r(s\vec{v}) = (rs)\vec{v}$. (b) Discuss multiplication in the system of vectors. What was your response to 1(d) above?
3. Writing vectors in component form: (x,y) and $x\vec{i} + y\vec{j}$, where the basic unit vectors $\vec{i}$ and $\vec{j}$ have end points (1,0) and (0,1). Recognizing that three-dimensional vectors in space follow the same rules as $x\vec{i} + y\vec{j}$ in the plane. The end points of the basic unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ are (1,0,0), (0,1,0), and (0,0,1), and $(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$. Finding the magnitude of vectors.	3. (a) Given point P(1,-2,3), give the coordinates of the point that is symmetric to P with respect to the (1) origin; (2) x-axis; (3) y-axis; (4) z-axis; (5) xy-plane; (6) xz-plane; (7) yz-plane. If vectors were drawn from the origin to each of the points above, how would their lengths compare? (b) Given points A(-2,3), and B(4,-5): find $\overrightarrow{AB}$ and $\lvert\overrightarrow{AB}\rvert$. Repeat for A(-1,2,1) and B(3,4,3).
4. Giving the direction vector, a vector equation, and parametric equations of a line, given two points.	4. (a) Give the vector equation and parametric equations of the line through the points A(-5,2) and B(1,5); (b) Repeat for A(3,2,3) and B(1,3,5).
5. Writing and sketching the equation of a plane Ax + By + CZ + D = 0.	5. In a three-dimensional coordinate system, sketch the graph of the plane: (a) x+y = 2; (b) 3x+2y+z = 6. In each plane, find the coordinates of the points, where the plane intersects the coordinate axes. (c) Find the set of points equidistant from the points (2,4,-1) and (0,5,6).

<table>
<tr>
<td>6. Defining the dot (inner) product, and, from the definition, developing the properties of the dot product;
using the dot product to determine the angle between two vectors and, in particular, to determine whether or not the vectors are parallel or perpendicular.</td>
<td>6. (a) Find angle θ included by vectors (4,-5,3) and (7,0,-1).
(b) Find k so that the vectors (2,-3,5) and (4,k,-1) are perpendicular.
(c) Find the angular relation between the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_1$, $\vec{v}_3$, and $\vec{v}_2$, $\vec{v}_3$: $\vec{v}_1=7\vec{i}-3\vec{j}+6\vec{k}$, $\vec{v}_2=3\vec{i}+3\vec{j}-2\vec{k}$, and $\vec{v}_3=6\vec{i}-16\vec{j}-15\vec{k}$.
(d) Find the points of intersection of the line $(x,y,z)=(3,4,-2)+t(2,-1,2)$ and the sphere $(x-3)^2+(y-4)^2+(z+2)^2=36$.</td>
</tr>
<tr>
<td>7. Applying vectors to proofs in geometry.</td>
<td>7. Prove, using vectors: The segment joining the midpoints of two sides of a triangle is one-half the length of the third side.</td>
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<tr>
<td>8. Defining and applying the cross product.</td>
<td>8. (a) Find the vector product for the following:
<table>
<tr><th>x</th><th>i</th><th>j</th><th>k</th></tr>
<tr><td>i</td><td></td><td></td><td></td></tr>
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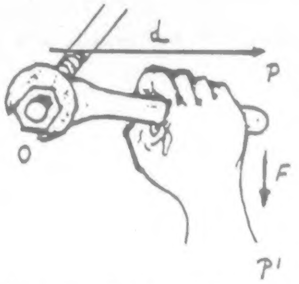

(b) The vector product, dxF, is called the moment of force about O.
$\overrightarrow{PP'}$ acts on $\overrightarrow{OP}$ to produce a moment of force about O. What is this moment if P(2,3,1) and P'(-2,6,4)?</td>
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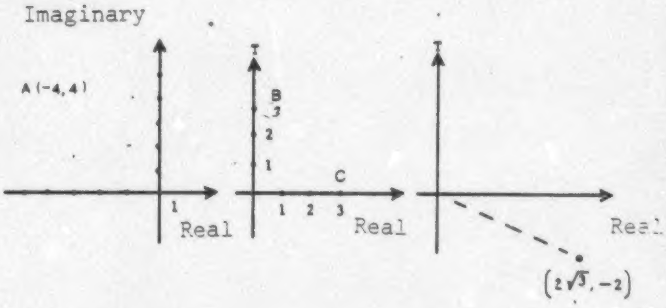
P. COMPLEX NUMBERS	
1. Recalling the fundamental operation with complex numbers.	1. Carry out the indicated operations, given: $z_1=2-3i, z_2=4+5i$; z_1+z_2; z_1-z_2; $\lvert z_1\rvert$; $\frac{z_1}{z_2}$; $z_1\bar{z}_1$; $z_2+\bar{z}_2$; $(z_2)^2$. Give each answer in simplified, standard form, a+bi.
2. Interpreting the graphic representation of z=a+bi in the complex plane by an Argand diagram; understanding the significance of z=a+bi, with $a\neq0$, $b\neq0$, z=a+0i, and z=0+bi; the relation of $r = \sqrt{a^2+b^2}$ and $\tan\theta = \frac{b}{a}$ and $a+bi = r(\cos\theta + i\sin\theta)$, or (a,b) versus (r,θ).	2. The graphs of points A,B,C,D are given; express each complex number in the form a+bi and $r(\cos\theta + i\sin\theta)$:  Imaginary A (-4, 4) 1 Real I B 3 2 1 C 1 2 3 Real I Real (2√3, -2)
3. Finding the product of two or more complex numbers and recognizing the role of the addition identities of the trigonometric functions; discovering the pattern evolved; computing powers of complex numbers.	3. $Z_1 = 2(\cos 30^\circ + i\sin 30^\circ)$; $Z_2 = 3(\cos 60^\circ + i\sin 60^\circ)$; (a) Compute Z_1Z_2. Write Z_1, Z_2 in standard form and compute Z_1Z_2. Compare results. (b) Multiply $(3, \frac{\pi}{3})$ by $(3, \frac{\pi}{4})$; (c) $Z = 2(\cos 120^\circ + i\sin 120^\circ)$; compute Z^3. (d) $Z = 1(\cos 60^\circ = i\sin 60^\circ)$; compute Z^2, Z^3, Z^4, Z^5, Z^6 and place the complex numbers on a unit circle $(1=1^2=1^3\ldots=1^6=r)$. Repeat for $Z = 1(\cos 30^\circ + i\sin 30^\circ)$; find Z^k, $k = 1,2,3,\ldots12$, and draw the graph of each. (e) How could you locate the vertices of a regular pentagon?

4. Appreciating the simplification of finding powers by De Moivre's theorem; proving or following the proof of the theorem.	4. If $Z = r(\cos\theta + i\sin\theta)$, then for any integer n, $Z^n = r^n(\cos n\theta + i\sin n\theta)$. From the exercises in (3.), we have observed a pattern. Using mathematical induction, we can easily show that propositions P_1, P_2, P_3 hold: n=1 $Z^1 = r^1(\cos 1\theta + i\sin 1\theta)$ n=2 $Z^2 = r^2(\cos^2\theta - \sin^2\theta + i(2\sin\theta\cos\theta)$ $= r^2(\cos 2\theta + i\sin 2\theta)$ n=3 $Z^3 = r^3(\cos 2\theta\cos\theta - \sin 2\theta\sin\theta + i(\sin 2\theta\cos\theta + \cos 2\theta\sin\theta) = r^3(\cos 3\theta + i\sin 3\theta)$, so P_3 holds. Fill in any steps omitted above, to be sure that you agree. Now we assume that all propositions $P_1, P_2, P_3, \ldots P_{k-1}$ hold, i.e., $Z^{k-1} = r^{k-1}(\cos(k-1)\theta + i\sin(k-1)\theta)$; we must show that $P_{k-1} \Longrightarrow P_k$. $P_k \ldots Z^k = Z^{k-1} \cdot Z$ $= r^{k-1} \cdot r(\cos(k-1)\theta + i\sin(k-1)\theta)(\cos\theta + i\sin\theta)$ $= r^k(\ldots\ )$ Can you complete the proof? Try.
5. Using the theorem for n negative and finding roots of complex numbers.	5. (a) $Z = r(\cos\theta + i\sin\theta)$; compute and simplify Z^{-1}. (b) Find the square roots of $Z = 4(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ and draw the graph. (c) Find the cube roots of -8.
6. Drawing graph of complex numbers $Z, Z^2, Z^3, \ldots,$ by joining points. (Remember, any power can be written in standard or polar form.)	6. $Z = 1+i$ Find, in two ways, (a,b) and (r,θ), $Z^2, Z^3, Z^4, \ldots, Z^{10}, \ldots$ Draw each complex number in the complex plane; connect the points by a smooth line. You should obtain a spiral, starting with Z at (1,1), and to Z^{10} ending at (0,32i). Is there an endpoint?

Q. PRACTICE

1. Solving equations and developing or refining techniques.	1. Find the solution set of each of the following: (a) $\frac{2x^2}{x+1} + \frac{3}{2} = \frac{2}{x+1}$ (b) $\begin{cases} 2x + y = 11 \\ x^2y^2+28xy = 480 \end{cases}$ (c) $\begin{cases} x^3 - y^3 = 1304 \\ x^2+xy+y^2 = 163 \end{cases}$ (d) $Z^2+(1-5i)Z-(12+5i)=0$.
2. Using determinants - cofactors or other simplification manipulations; finding areas, using coordinates of vertices.	2. (a) Using cofactors, evaluate $\begin{vmatrix} 3 & 1 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}$ (b) $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & j \end{vmatrix} = (\ ) \cdot \begin{vmatrix} e & h \\ f & j \end{vmatrix} - (\ ) \begin{vmatrix} d & g \\ f & j \end{vmatrix} + (\ ) \cdot \begin{vmatrix} d & g \\ e & h \end{vmatrix}$ (c) Solve the system $\begin{cases} x + y + 2z = 2 \\ 2x - y + 3z = 5 \\ x - y - z = -2 \end{cases}$ (d) The vertices of ΔABC are at (x_1,y_1), (x_2,y_2), (x_3,y_3); draw a graph and find the area using rt.Δ's or trapezoids. Show that your result is equal to $K = \frac{1}{2}\left(\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \right)$, or $K = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$, where K is area. (e) Find the area of the irregular polygon with vertices at (1,2), (5,1), (9,3), and (8,9); (draw diagonals) and repeat above.

3. Using graphs when algebraic solutions fail.	3. (a) Find the zeros of $F:x\to e^x+x^2-4x^2$. $e^x+x^4-4x^2=0$ iff $e^x=4x^2-x^4$. (Hint: let $f(x)=e^x$ and $g(x)=4x^2-x^4$.) Solve graphically to nearest tenth; check, if possible, by calculator. (b) On the interval $-3\pi \le x \le 3\pi$, draw the graph of $y = x\cos x$ and determine the absolute maximum and minimum; (draw the graphs of $y = \pm x$, $y = \cos x$ in the same coordinate system and multiply ordinates).
4. Working on individual projects, interesting programs, and solutions to some of the exercises, depending on the student's preparation in computer science.	4. The area of a Δ is $K=\frac{1}{2}ab \sin \gamma$. (a) A regular polygon of n sides can be divided into n Δ's, each of area $\frac{1}{2} r^2 \sin \frac{2\pi}{n}$; let $r = 1$; the area of the polygon will then be $\frac{n}{2} \sin \frac{2\pi}{n}$. Check this. As n = 3, 4, 5, 6, compute this product. Write a program that will give a list of values as n increases. What should $\lim_{n\to\infty} \frac{n}{2} \sin \frac{2\pi}{n}$ be? Draw a diagram. (b) Write a program that will give the value of e, using: $e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ or $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.

R. ADVANCED MATHEMATICS AND CAREERS

Investigating one job from each of three occupational clusters in which the skills learned in Advanced Mathematics can be applied.

DIFFERENTIAL AND INTEGRAL CALCULUS OBJECTIVES

Introduction

The objectives listed in this document are an outline for the course requirements, with the illustrating examples pointing out the depth of understanding and learning expected. The particular order or sequence of the objective sections represent one approach to calculus instruction. The mathematical knowledge that students bring to the course will indicate how much time must be given to the topics of Analytic Geometry, Trigonometry, and Elementary Functions, and lead to the choice between Calculus AB or Calculus BC.

Once these important factors are decided, however, there is little room for flexibility. The objectives are based on The Advanced Placement Course Description, Calculus AB, Calculus BC, May 1983 and May 1984, published by the College Board, which is the recognized syllabus for this subject area and the basis of the AP exams.

Calculus AB covers Elementary Functions and more than half the academic year must be given to the study of calculus. Calculus BC is a full year calculus course; in addition to all the topics required for Calculus AB, it includes two more topics: Sequences and Series; and Differential Equations. Throughout the objectives, topics and examples meant for Calculus BC only are marked by an asterisk.

With calculators and computers on the scene, some topics previously not accessible to thorough analysis take on new dimensions. Students, with their computer course behind them, can add much to their depth of understanding, working independently and sharing results with the class. The first concern, however, must be to meet all the objectives listed, or we fail to give students the opportunity to measure up to the challenge of the AP Examination.

A note on the Questions for Pre-Calculus section: objectives for the calculus course are, of necessity, rooted in the mathematics skills and concepts learned learlier. Calculus is the course that unified and summarizes all previous course work. The purpose of the first four pages of questions with solutions is to help teachers evaluate students' preparation and point out the importance of the background knowledge that students must bring to the course. It should also highlight the importance of careful, dtailed solutions. It is only a short survey; more study and review is presented in the first sections, with familiar topics interwoven as the course develops.

Students exposed to calculus should experience the beauty and genius of a creation that represents one of the greatest achievements of the human mind. The credit earned by this college-level course will, in all probability, meet college mathematics requirements and will therefore allow more time and greater choice in college. The demands of the course, AB or BC, as well as those of any calculus course derived from these objectives, are great - but so are the rewards.

The Calculus objectives are divided into the following sections:

Questions for Pre-Calculus

A. Elementary Functions

B. Differential Calculus

C. Integral Calculus

D. Sequences and Series

E. Differential Equations

F. Calculus and Careers

QUESTIONS FOR PRECALCULUS

The following are ADVANCED PLACEMENT MATHEMATICS EXAMINATION QUESTIONS, requiring no knowledge of calculus.

1973: AB 1: Given $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = 4$:

(a) Find the coordinates of the points common to the graphs of f and g.

(b) Find all the zeros of f.

(a) $x^3 - 6x^2 + 9x = 4$

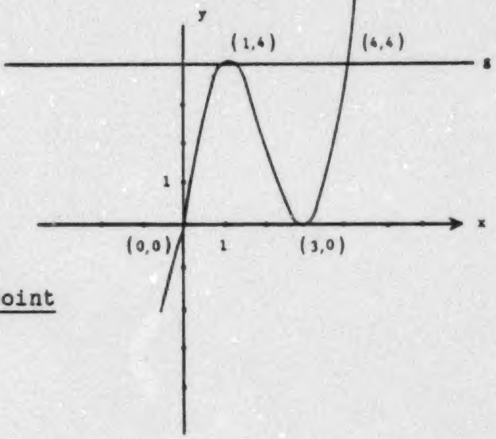
$\Longrightarrow x^3 - 6x^2 + 9x - 4 = 0$

$\Longrightarrow (x - 1)(x^2 - 5x + 4) = 0$

$\Longrightarrow (x - 1)(x - 1)(x - 4) = 0$

$x = 1,\ y = 4;\ \underline{(1,4)}\ \underline{\text{double point}}$

and $x = 4,\ y = 4;\ \underline{(4,4)}$



(b) $f(x) = x(x - 3)^2$ zeros: $\underline{0,\ 3\ \text{mult.}\ 2}$

AB 4: What is the set of all values of b for which the graphs of $y = 2x + b$ and $y^2 = 4x$ intersect in two distinct points?

$4x = 2y - 2b = y^2$

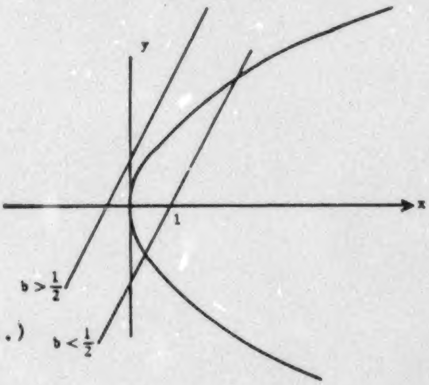
$\therefore\ y^2 - 2y + 2b = 0$

Two distinct points of intersection if $D > 0$.

$4 - 8b > 0$

$\Longrightarrow \underline{b < \tfrac{1}{2}}$

(For $b = \frac{1}{2}$, $D = 0$, line is tangent.)



PRECALCULUS 1974

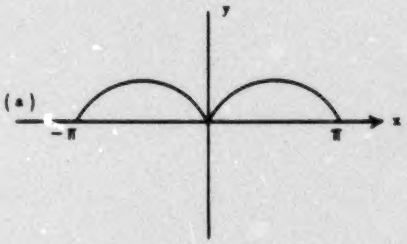
1974: AB 1: Given $f(x) = |\sin x|$, $-\pi \leq x \leq \pi$, and $g(x) = x^2$ for all real x:

(a) On the axes provided, sketch the graph of f.

(b) Let $H(x) = g(f(x))$. Write an expression for $H(x)$.

(c) Find the domain and range of H.

(a)



(b) $\underline{H(x)} = (f(x))^2 = \underline{\underline{\sin^2 x}}$

(c) $\underline{\text{Domain}}$ of H: $\underline{\{x \mid -\pi \leq x \leq \pi\}}$

$\underline{\text{Range}}$ of H: $\underline{\{y \mid 0 \leq y \leq 1\}}$

1974: AB 6: Given two functions f and g defined by $f(x) = \tan x$ and $g(x) = \sqrt{2}\cos x$:

(a) Find the coordinates of the points of intersection of the graphs of f and g in the interval $0 < x < \frac{1}{2}\pi$.
Solution:

$\tan x = \sqrt{2}\cos x$ and $\cos x \neq 0$.

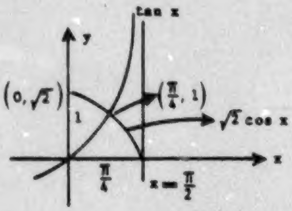
$\sin x = \sqrt{2}\cos^2 x$

$\sin x = \sqrt{2}(1 - \sin^2 x)$

$\sqrt{2}\sin^2 x + \sin x - \sqrt{2} = 0$

$(\sqrt{2}\sin x - 1)(\underset{\neq 0}{\sin x + \sqrt{2}}) = 0$ iff $\sqrt{2}\sin x - 1 = 0 \Rightarrow x = \frac{\pi}{4}$

$\underline{(\frac{\pi}{4}, 1)}$.



PRECALCULUS 1975,1976

1975: AB 1: Given the function f defined by $f(x) = \ln(x^2-9)$:

(a) Describe the symmetry of the graph of f.

(b) Find the domain of f.

(c) Find all values of x such that $f(x) = 0$.

(d) Write a formula for $f^{-1}(x)$, the inverse function of f, for $x > 3$.

(a) $f(-x) = \ln((-x)^2-9) = \ln(x^2-9) = f(x)$.

$\therefore$ $\underline{\text{symmetric about y-axis}}$.

(b) $x^2 - 9 > 0 \Longrightarrow |x| > 3$

$\underline{\{x \mid x < -3 \text{ or } 3 < x\}}$

(c) $f(x) = 0 \Longrightarrow x^2 - 9 = 1$

$$|x| = \sqrt{10}$$

$$\underline{x = \sqrt{10} \text{ or } -\sqrt{10}}.$$

(d) $y = \ln(x^2-9)$ and $(x,y) \rightarrow (y,x)$

$x = \ln(y^2-9)$

$y^2 - 9 = e^x$

$y^2 = e^x + 9$ and $x > 3$

$y = \underline{f^{-1}(x) = \sqrt{e^x+9}}$

1976: AB 1: Let f be a real-valued function defined by $f(x) = \sqrt{1+6x}$.

(a) Give the domain and range of f.

$1 + 6x \geq 0$ and $f(x) \geq 0$ $\therefore$ domain: $\underline{\{x \mid x \geq -\frac{1}{6}\}}$;

range: $\underline{\{y \mid y \geq 0\}}$.

PRECALCULUS 1976

AB 2: Given two functions f and h such that

$$f(x) = x^3 - 3x^2 - 4x + 12 \text{ and}$$

$$h(x) = \begin{cases} \dfrac{f(x)}{x-3}, & \text{for } x \neq 3 \\ p, & \text{for } x = 3 \end{cases}$$

(a) Find all zeros of the function f.

(b) Find the value of p so that function h is continuous at $x = 3$.

(c) Using the value of p found in (b), determine whether h is an even function. (Note: intentionally omitted in (b) and (c); 'justify your answer.')

(a) $f(x) = x^2(x-3) - 4(x-3)$

$= (x+2)(x-2)(x-3)$ zeros: $-2, 2, 3$.

(b) $h(x) = x^2 - 4$, for $x \neq 3$; $h(3) = 9 - 4 = 5$

$\therefore p = h(3) = 5$

Let $p = 5$, then h continuous at $x = 3$.

(c) $h(x) = x^2 - 4$ for $x + 3$ and $h(x) = 5$ for $x = 3$

$h(-x) = (-x)^2 - 4 = x^2 - 4$ for all x

$h(-x) = h(x)$, h is even.

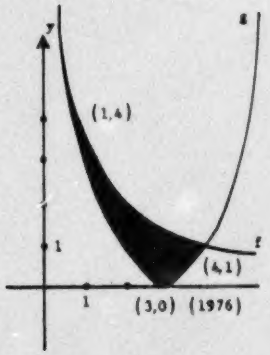
PRECALCULUS 1976,1977,1979

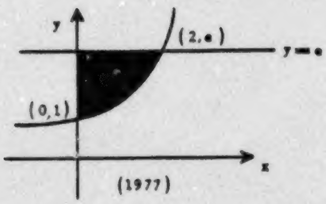
Draw the graphs of the regions bounded by the graphs of:

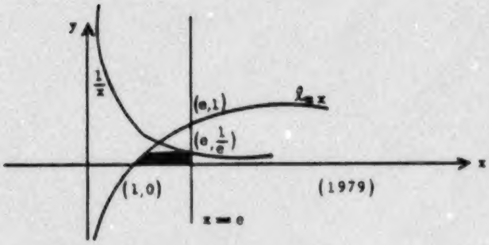
1976: AB 3: $f(x) = \frac{4}{x}$ and $g(x) = (x-3)^2$.

1977: AB 3: $f(x) = e^{\frac{1}{2}x}$, the line $y = e$, and the y-axis.

1979: AB 5: $y = \frac{1}{x} \ln x$, the x-axis and the line $x = e$.


y
g
(1,4)
1
f
(4,1)
1
(3,0)
(1976)

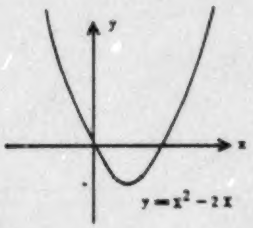

y
(2,e)
y = e
(0,1)
x
(1977)

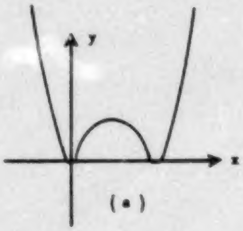

y
$\frac{1}{x}$
(e,1)
ln x
(e, $\frac{1}{e}$)
x
(1,0)
(1979)
x = e

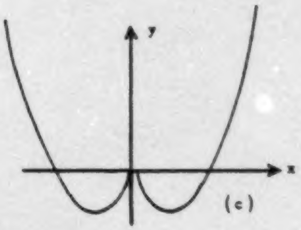
PRECALCULUS 1979,1980

1979: AB 6: The curve in the figure represents the graph of f, where $f(x) = x^2 - 2x$. On the axes provided, sketch the graph of

(a) $y = |f(x)|$ and (c) $y = f(|x|)$.







1980: AB 3: Let $f(x) = \ln(x^2)$ for $x > 0$ and $g(x) = e^{2x}$ for $x \geq 0$. Let H be the composition of f with g, that is $H(x) = f(g(x))$, and let K be the composition g with f, that is $K(x) = g(f(x))$.

(a) Find the domain of H and write an expression for H(x) that does not contain the exponential function.

(b) Find the domain of K and write an expression for K(x) that does not contain the exponential function.

(c) Find an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f, and find the domain of f^{-1}.

(a) $H(x) = f(g(x)) = \ln(e^{2x})^2 = \ln e^{4x} = 4x$

$$g(x) > 0 \ \forall \ x \geq 0 \ \therefore \ \underline{H(x) = 4x \text{ for } x \geq 0}$$

(b) $K(x) = g(f(x)) = e^{2\ln x^2} = e^{\ln x^4}$

$$\ln K(x) = \ln x^4 \ln e = \ln x^4 \ \therefore \ \underline{K(x) = x^4}$$

$$\ln x^2 \geq 0 \text{ for } x \geq 1 \ \therefore \text{ domain } \underline{\text{all } x \geq 1}.$$

(c) $y = \ln(x^2)$, $x > 0$ and y is all reals.

$$(x,y) \rightarrow (y,x) \Longrightarrow x = \ln(y^2) \text{ and } y^2 = e^x \Longrightarrow y = e^{\frac{x}{2}}.$$

$$\therefore \ \underline{f^{-1}(x) = e^{\frac{x}{2}}}, \ \underline{x \in \text{reals}}.$$

PRECALCULUS 1981

1981: AB 4: Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$.

(a) Is f an even function? Justify your answer.

(b) Find the domain of f.

(c) Find the range of f.

(a) $f(-x) = 5^{\sqrt{2(-x)^2-1}} = 5^{\sqrt{2x^2-1}} = f(x)$ $\therefore$ $\underline{f \text{ is even}}$.

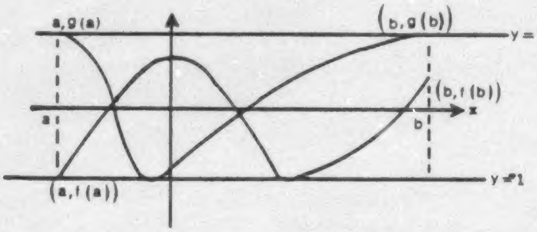
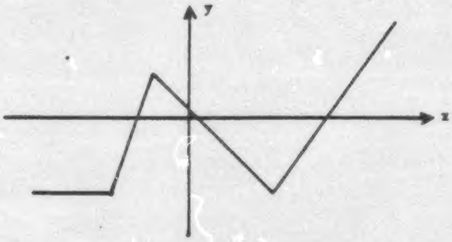
(b) $2x^2 - 1 \geq 0$ Domain of f: $\left\{\underline{x \mid x \leq -\frac{2}{2} \text{ or } \frac{2}{2} \leq x}\right\}$.

$x^2 \geq \frac{1}{2}$

(c) $f(x) \geq 5^0 = 1$ Range of f: $\left\{\underline{\ y \mid y \geq 1\ }\right\}$ or

$[1, \infty)$.

DIFFERENTIAL AND INTEGRAL CALCULUS

NOTE: * BC Only A. **ELEMENTARY FUNCTIONS**	
1. Recognizing, analyzing, applying the properties, and drawing the graph of elementary functions: algebraic, trigonometric, exponential, and logarithmic; stating the domain, range, writing the equation of the inverse, and drawing the graph.	1. If f is defined by the equation $f(x) = \sqrt{1-5x}$, give the domain, range, the x- and y-intercepts; write the equation of f^{-1}, the inverse of f, and draw the graph of f and f^{-1}.
2. Recognizing the relation between the domain and range of functions and the domain and range of their composition. Gaining familiarity and using both notations: the mapping, $f:x \to e^x$, and the function defined by $f(x) = e^x$.	2. (a) Given the functions $f:x \to \ln x$, and $g:x \to x^2 - 2$; if $F = fg$, and $G = gf$, give the domain and range of F and G. (b) If $g:x \to 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$, find f.
3. Handling functions involving sums, products, quotients, absolute values of functions; drawing graphs by adding, multiplying, taking reciprocals of ordinates.	3. (a) If $k > 1$ is a constant, and $x > 0$, how does the graph $\ln kx$ compare to the graph of $\ln x$? State and illustrate by a graph. (b) Given two graphs, f and g, on [a,b], sketch the graph of $f \cdot g$.  a,g(a) (b,g(b)) y=1 (b,f(b)) x a b y=-1 (a,f(a)) Given the graph of g, draw the graph of $\lvert g(x) \rvert$, and $g(\lvert x \rvert)$. 

CALCULUS A

*4. Understanding and working with parametrically defined functions.	4. In each of the following, write the Cartesian equation, indicate domain and range, and draw the graph. (a) $x = t^2$, $y = t^3$; (b) $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$, $t > 0$; (c) $x = \cos 2\theta$, $y = \cos \theta$.
5. Stating, in terms of the graph of the function as well as algebraically, the symmetry; zeros; whether the function is odd or even.	5. (a) Given the polynomial function, $f: x \to x^2(x^2-4)$, for $\lvert x \rvert \leq 3$; give the range, zeros, symmetry, and sketch the graph. (b) From the graph above, draw the graph of $f + g$, where $g: x \to 4$. Discuss the changes, comparing answers to (a). (c) Explain, by giving a function and its graph, the meaning of $f(-x)=f(x)$; $f(-x)=-f(x)$; and for every point (a,b) on the graph of f, there is a point (b,a) on the graph of h. (d) The graph of $y = ax^4+bx^3+cx^2+dx+e$, $a \neq 0$, is symmetric about the y-axis; what can you tell about the coefficients? What is the name of such a function?
6. Analyzing restrictions in domain and equations of asymptotes.	6. (a) Give the domain and range, symmetry, equations of asymptotes, and extent of $y = x + \frac{1}{x^2}$ (b) If $f(x) = x$, and $g(x) = \frac{1}{x^2}$ in the graph of $f + g$ above, which graph dominates for $\lvert x \rvert \to 0$? $\lvert x \rvert \to \infty$?

CALCULUS A

7. Recognizing even and odd functions; understanding the concepts of periodicity, amplitude, horizontal shift.	7. (a) Which of the following functions are odd? Which are even? $y = \sin 2x$; $y = \cos 3x$; $y = \tan x$; $y = x$; $y = x^4$; $y = 5\|x\|$. (b) Give the period of each of the following: $f(x+a) = f(x)$: $\sin x$; $\cos x$; $\tan x$; $\sin 2x$; $\cos \frac{1}{2}x$; $\tan 3x$. (c) Give the amplitude of $f(x) = \sqrt{3} \sin x + \cos x$; write f in terms of cosine only. (d) Give the amplitude, period, and horizontal shift of $y = 3 \sin(2x - \frac{\pi}{3})$.
8. Using trigonometric identities; addition, multiple, and half-angle formulas; properties of logarithmic and exponential functions.	8. (a) Find the zeros of f, if $f(x) = \sin 2x + \sqrt{2} \sin x$, $0 \leq x < 2\pi$. (b) Write in terms of sin 3x: $y = \sin 3x \cos^2 3x$. (c) Give the domain, range, and zeros of $f(x) = y = \ln(\cos x)$, $0 \leq x < 2\pi$; draw the graph and discuss the symmetry of f. (d) Draw the region enclosed by the graphs of $y = e^x$, $y = e^{-x}$, and $y = 3$.

	Give the coordinates of the points of intersection and write an expression for the area of the triangle formed by joining these points with line segments. Write an expression for the volume generated, if the triangle rotates about the y-axis.
9. Understanding the relation between the functions: a^x and $\log_a x$, and corresponding graphs.	9. Given the graph of $f: x \rightarrow \ln x$ (i.e., $f(x) = \ln x$), with the segment joining point A, the x-intercept, and point $B(e^2, f(e^2))$; (a) write the slope of $\overleftrightarrow{AB}$; (b) write the equation of the line through the points on the graph of f^{-1}, corresponding to points A and B. (c) Draw the graph of $y = e^{-x^2}$; check symmetry, intercepts, write equation of asymptote(s).
10. Realizing that the basic concept of calculus, its very foundation, is that of limit; understanding, and stating (without proof), the laws of limits, and applying them to evaluate the limit of an expression.	10. Evaluate each of the following limits: $\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1}$; $\lim\limits_{x \to 5} 2$; $\lim\limits_{x \to 2} \lvert x-2 \rvert$; $\lim\limits_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \geq 1; \end{cases}$ sketch the graph. $\lim\limits_{x \to 2^-} \frac{x^2 - 4}{x - 2}$; $\lim\limits_{x \to 2^+} \frac{x^2 - 4}{x - 2}$; draw a conclusion and give your reason for the existence of $\lim\limits_{x \to 2} \frac{x^2-4}{x-2}$, and sketch the graph.

*11. Using the ε-δ method to prove continuity at a point, or evaluate δ, or find $M > 0$, in proving a limit problem.	11. (a) Find, for $\varepsilon > 0$, $\delta > 0$, to verify that $\lim_{x \to 2} (x^2-x)=2$. (b) Prove or disprove the existence of $\lim_{x \to \infty} \frac{x+1}{x-1}$; sketch a graph. (c) The area of a regular polygon of n sides, inscribed in a circle of radius r, is given by $K = \frac{n}{2}r^2 \sin \frac{2\pi}{n}$. Find $\lim_{n \to \infty} K$ and discuss the result.
12. Stating the definition of continuity: f is continuous at $x = a$ iff (a) $f(a)$ exists (b) $\lim_{x \to a} f(x)$ exists, and (c) $\lim_{x \to a} f(x) = f(a)$. Applying the theorems of continuity, no proof required; differentiating between removable and nonremovable discontinuity; locating the maximum and minimum of a function, continuous on [a,b]; using the intermediate value theorem.	12. (a) In the following examples state and explain why a function is or is not continuous; if there is a discontinuity, is it removable? $f(x) = \begin{cases} 3 + 2x, & x \leq 3 \\ x^2, & x > 3 \end{cases}$; $f(x) = \frac{x + 2}{x^2-3x-10}$; sketch the graphs of both functions above.

(b) Find the value of k so that the function, defined by

$$f(x) = \begin{cases} x^3 + 2x - 2, & x \le 1 \\ k|x| + 3, & x > 1 \end{cases},$$

is continuous for all x. If the above function is defined only for $[-3,4]$, give its maximum and minimum values on the interval: find all real numbers c, $-3 \le c \le 4$, such that $f(c) = -4$. A careful graph will check, in fact, provide, all answers.

B. DIFFERENTIAL CALCULUS

1. (a) Understanding that the derivative, if it exists, is

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\left(\text{At point a, } f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}\right).$$

(b) Understanding that the derivative denotes both the slope of the graph of f at the point $(x,f(x))$ and the instantaneous rate of change of $f(x)$ with respect to x.

(c) Compute $\frac{\Delta y}{\Delta x}$, and understand the relation between the slopes of a secant line and the tangent line.

(d) Compute $\frac{\Delta s}{\Delta t}$, and understand the relation between average velocity and instantaneous velocity.

1. (a) Given $f(x) = x^3 - 4x$, use the definition of the derivative at a point, $x = a$, to find $f'(a)$ and compute $f'(a)$ for $a = -2, -1, 0, 1, 2$.

(b) Use the definition of the derivative, to express

$$\frac{\Delta y}{\Delta x} \text{ and } \frac{dy}{dx}, \quad \text{given } f(x) = \sqrt{2x-3}.$$

State the domain of f and f'; find the slope of the secant, if $x = 2$, and $\Delta x = 12$; if $A(2,1)$ and $B(14,5)$; write the equation of the tangent to the graph of f, at $x = 2$; at $x = 14$; find the tan α, where α is the measure of the angle between the tangent and the secant at point A; estimate α.

(c) A particle moves according to the given rule; find the average rate of change between the two given points and compare to the instantaneous rate of change at each point:

$$s(t) = 2t^2 + 7t \qquad s(t) = \frac{1}{1+t},$$

$$(1,9),\ (2,22); \qquad (0,1),\ (3,\tfrac{1}{4}).$$

2. Using the rules for differentiation of constant, power, sum, product, and quotient; applying to problem solving.

2. (a) Find the derivative of each of the following:

$y = 2x^3 + 3x^2 - 5x + 1;$

$y = \sqrt{x^3} - \frac{2}{x^3};$

$y = x^3(2x+5)$; use two methods;

$$y = \frac{2x^2 + 3x - 5}{x^2 - 2x}.$$

(b) If acceleration is $\frac{dv}{dt} = \frac{d^2s}{dt^2}$ the second derivative of s with respect to t, find the velocity and acceleration of an object, whose position s at time t is

$s = t^2 + 4t + 5;$

$s = (2t + 3)^3.$

(c) As blood moves from the heart through the major arteries to the capillaries and back through the veins, the systolic pressure drops. If this pressure is given by

$$p = \frac{25t^2 + 125}{t^2 + 1},$$

p in millimeters of mercury, t in seconds, at what rate is the pressure dropping 5 seconds after leaving the heart? (Use a calculator to answer to the nearest hundreth mm/sec.)

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3. Stating and applying the chain rule for composite functions.

3. (a) Expand and find the derivative of $y = (3x^2 + 4)^3$; repeat, writing $y = u^3$, and $u = 3x^2 + 4$.

(b) Identify in each case two functions, f and g, and find h'(x), if $h(x) = g(f(x))$.

CALCULUS B

	$h(x) = \frac{1}{\sqrt{x^2+25}}$; $h(x) = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ $h(x) = \cos(x^2+1)$ and $\frac{d(\cos x)}{dx} = -\sin x$. (c) If $\frac{d(\sin x)}{dx} = \cos x$, differentiate each of the following: $f(x) = \sqrt{x}\sin x$; $f(x) = \sin\sqrt{x}$; $f(x) = x^{\frac{3}{2}}\tan x$.
4. Differentiating implicitly defined functions.	4. (a) Find $\frac{dy}{dx}$, given $x^2 + y^2 = 25$; draw the radius of the circle to P(3,4) and evaluate the slope of the normal at P; evaluate $\frac{dy}{dx}$ at (3,4); explain. (b) Find $\frac{dy}{dx}$, given $x^3 - xy + y^2 = 4$; $\frac{x^2}{25} - \frac{y^2}{9} = 1$; (c) Given: $2x^2 + y^2 = 6$ and $y^2 = 4x$. Find the slope of each curve and show that the graphs are orthogonal (i.e., they intersect at right angles); sketch the graph.
5. Making a survey of the functions studied, recognizing the class of algebraic versus transcendental functions; finding derivatives of all circular functions, including Arc sin x, ($\mathrm{Sin}^{-1}x$), and Arc tan x, ($\mathrm{Tan}^{-1}x$).	5. (a) Find the derivative of each of the following: $y = 2\sec^3 x$; $y = \tan^2(4x)$; $y = 2\sin^2(3x^3)$; $y = \mathrm{Sin}^{-1}(x^2)$; $y = \mathrm{Arc}\tan\left(\frac{3}{x}\right)$; $y = x\,\mathrm{Arc}\cos(2x) - \frac{1}{2}\sqrt{1-4x^2}$. (b) What function do you get for $f'(x)$, if $f(x) = ax^n + bx^{n-1} + \ldots$; $a \neq 0$, $n \in N$; if $f(x) = \frac{ax+b}{cx+d}$? $a,b,c,d \in R$, $\neq 0$. if $f(x) = \mathrm{Sin}^{-1}(x)$? $\mathrm{Tan}^{-1}x$?

CALCULUS B

*6. Finding the derivative of parametrically defined, and vector functions: applying derivatives to polar equations and their graphs.	6. (a) Write the equation of the tangent to the graph of $x = 2(\theta-\sin\theta)$, $y = 2(1-\cos\theta)$, at $\theta = \pi$. (b) A particle moves along the circle $x^2+y^2=1$; at time $t \geq 0$, the position vector is given by the vector $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$; find the velocity vector. If t increases without bound, what point does the particle approach? (c) Sketch the graph of $r = 2 \sin 3\theta$ and find the slope of the curve at $\theta = \frac{\pi}{6}$; $\theta = \frac{\pi}{4}$. (d) Sketch the graph of $r = a(1+\cos\theta)$; find the points on the curve where the tangent is parallel to x-axis, to y-axis.
7. Appreciating the "geometric" derivation of $\lim_{\theta\to 0} \frac{\sin\theta}{\theta}$, and the definition of the number $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$; its role in finding the derivative of ln x, using the Δ-method; differentiating logarithmic functions and applying logarithmic differentiation to nonlogarithmic functions, as an aid.	7. (a) Find the derivative of $y = \ln(x^2-3)$; $y = \ln e^x$; $y = \frac{\ln x}{x}$; $y = x \ln x$; $y = \sqrt{2x+5}$; $y = \ln(x^2+4) - x\,\mathrm{Tan}^{-1}(\frac{x}{2})$. (b) Find the derivative of $y = x^{x-1}$; $y = x^{\ln x}$; $y = (x^2+4)^4(x^2-3)^{\frac{1}{4}}$. (c) Find the derivatives of $y = \ln x$, $y = \log_e 2x$, $y = \ln 5x$, $y = \ln nx$; explain your answers. (d) Find $\frac{dy}{dx}$, given: $\ln(x+y) = \mathrm{Tan}^{-1}(\frac{x}{y})$.

CALCULUS B

8. Differentiating exponential functions; recognizing that the derivative of any order of $y = e^x$ is e^x, and that any real number $a > 0$ can be expressed in terms of e: $a = e^{\ln a}$.	8. Differentiate: $y = e^{2x-1}$; $y = e^{-2x} + \frac{e^2}{x^3}$; $y = 2^{4x}$; $y = 3^{x^2}$; $y = xe^{\tan x}$; $y = x^3e^x-2xe^x+2e^x$.
9. Expressing derivatives of higher order.	9. (a) Find the first, second, third, and fourth derivative of: $y = x^4-3x^3+2x-5$; $y = -\frac{1}{x}$; $y = \sin x$; $y = e^{3x}$ (b) If $n \in N$, N = {positive integers}, how many nonzero derivatives does x^n have? If $n \in Q$, Q = {rational numbers}, how many distinct derivatives does cos x have? sin x?
10. Understanding, stating, and applying Rolle's theorem; recognizing its importance in proving the mean value theorem; stating (without proof) and interpreting graphically the mean value theorem.	10. (a) Which of the following functions meet the conditions for Rolle's theorem on the given interval? Explain your answer in each case. $f: x \rightarrow x^2+x-6$, $[-3,2]$; $g: x \rightarrow 3-\|x-5\|$; $[2,8]$; $h: x \rightarrow e^x-1$, $[-2,2]$. (b) Find an interval on which Rolle's theorem can be applied, and find all values of c for which $f'(c)=0$. $f(x)=x^3-4x$; $f(x)=\cos x$, for $\frac{-5\pi}{2} < x < \frac{5\pi}{2}$.

CALCULUS B

	(c) Find all values of c for which the tangent to the curve is parallel to the secant, joining the points on the curve at x=a and x=b, given [a,b]; $f(x) = x^3-3x^2-10x+24$, [-4,5]; $f(x) = \sqrt{2x-5}$, [7,15]; $f(x) = x^{\frac{2}{3}}$, [-1,1]; $f(x) = x^{\frac{2}{3}}$, [0,1].
11. Understanding the concepts of increasing and decreasing function, and the relation of the sign of the first and second derivative; recognizing the behavior of the graph and applying to locating critical points.	11. (a) Give the interval for which the function $f(x) = x^2(x^2-4)$ is increasing; is decreasing; find the coordinates of the points on the graph, where the slope is zero. (b) Repeat the above for $f(x)=x^3-x^2-5x$; locate the critical points; approximate the zeroes, and sketch the graph. (c) Find a and b, if $y = x^3+ax^2-3x+b$ has a point of inflection at (1,1).
12. Recognizing the relation between continuity and differentiability; stating the definition of both concepts and noting again that each is defined in terms of a limit.	12. (a) True or False? A function that is continuous at a point is also differentiable. ______ A function that is differentiable at a point is also continuous. ______ (b) In each of the following give the set of points x in R, for which the function is continuous, (A), for which it is differentiable, (B); write an expression for f',(C).

Function $f(x)$	A	B	C
x^2+3x+2			
$\lvert x-2 \rvert+3$			
$\ln(x-3)$			
$\dfrac{x^2+3x}{x^2+2}$			
$x^{\frac{2}{3}}$			
$\dfrac{x^2-9}{x+3}$			

13. Stating and applying l'Hopital's rule; recognizing when it will be used; using care with proper notation when evaluating indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$; noting that problems will arise if hypotheses are not met (see b).

13. (a) Evaluate the limits of: $\lim\limits_{x\to 3} \dfrac{2x^2-18}{x-3}$; $\lim\limits_{x\to\infty} \dfrac{x(x+2)}{x^2}$;

$\lim\limits_{\theta\to 0} \dfrac{\sin 3\theta}{\theta}$;

which indeterminate form is represented in each case? Do you need and can you use l'Hopital's rule?

(b) Evaluate the following limits:

$\lim\limits_{x\to 0} \dfrac{\sin x}{x+1}$, $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\tan x}{\sec x}$;

explain, if anything special happened.

(c) Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$;

$\lim\limits_{x\to 0} \dfrac{\sin 3x}{\sin x + \sin 4x}$.

*14. Applying l'Hopital's rule to indeterminate forms other than $\frac{0}{0}$ and $\frac{\infty}{\infty}$.

14. Evaluate each of the following limits:

$\lim\limits_{x\to 2} \left(\dfrac{1}{x^2-4} - \dfrac{\sqrt{x-1}}{x^2-4}\right)$; $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$;

$\lim\limits_{x\to \frac{\pi}{2}} (x \tan x - \frac{\pi}{2} \sec x)$;

$\lim\limits_{x\to \frac{\pi}{4}} (1 - \tan x) \sec 2x$.

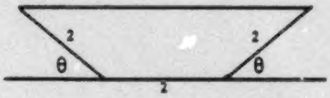
15. Applying the techniques in this section to various problems, i.e.,
    curve tracing,
    concavity,
    maxima-minima problems, and
    related rates

15. (a) Given $f: x \rightarrow 3x^4 - 4x^3$, $-2 \leq x \leq 2$, find the zeros of f; find the intervals for which $f'(x) < 0$, and for which $f'(x) > 0$; the intervals on which the graph is concave up, concave down; find the coordinates of all maximum, minimum, and inflection points. Use all the information and sketch the graph. (Use 1 unit on the x-axis = 10 units on the y-axis.)

(b) Given the function defined by $y = x - \frac{4}{x}$, give the set of values of x for which the graph of f is:
   (1) increasing
   (2) decreasing
   (3) concave upwared
   (4) concave downward
What happens to the graph of f if $x \rightarrow \pm \infty$? Draw the graph.

(c) Find the shortest distance from the point (1,2) to the parabola $y = \frac{x^2}{4}$.

(d) For what angle θ does the trapezoid have maximum area?



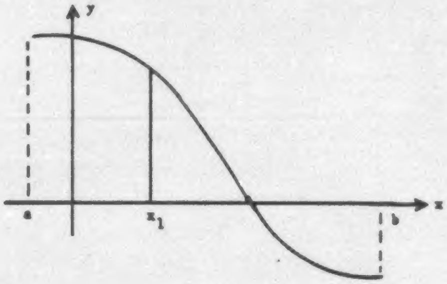
(e) The electric power P in watts in a d.c. circuit with two resistors of resistance R_1 and R_2 connected in series is

$$p = \frac{v R_1 R_2}{(R_1 + R_2)^2},$$

where v is the voltage. If v and R_1 are held constant, what resistance R_2 produces maximum power?

	(f) The volume of a sphere is decreasing at the rate of 12π ft^3/min. Find the rate at which the radius is changing at the instant when the radius is 20 ft. (g) A point moves along a parabola whose equation is $4y = x^2$, so that the x-coordinate increases at the constant rate of 2 units per second. At what rate is the distance between this point and the point (0,4) changing, when $x = 2$? Draw a graph that explains the answer. (h) A tank has the shape of a circular cone. Its depth is 30 ft, and the radius at the top is 15 ft. The tank is being filled in such a way that the water level increases at a constant rate of 4 ft/hr. At what rate is water entering the tank at the moment the water level is at 8 ft?
16. Stating the definition of differential; understanding the change in notation from the nonseparable symbol, $\frac{dy}{dx} = f'(x)$, to differential notation, without leading to any inconsistency; applying differentials to linear approximations and evaluations of errors; using also Newton's method for approximation of zeros of functions; including graphic interpretation.	16. (a) If u and v are differentiable functions in x, express, using differentials, d(uv); express $d(\sin 2x)$; $d(\cos^2 3x)$; $d(e^{3x})$. (b) Use differentials to approximate $\sqrt[3]{126}$. (c) The length of the edge of a cube is 12 cm, with a possible error in measurement of 0.03 cm; using differentials, approximate the maximum possible error in the volume of the cube, in the surface area.

(d) Show graphically how an approximation, x_2, would improve the first guess of x_1, using Newton's method; repeat for x_3.



(e) Use a calculator and Newton's method to find the zero of $f(x) = x^3-3x-1$ (to three decimal places), in the interval [-1,0].

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*17. Applying velocity vectors, acceleration vectors to motion in the plane; stating and applying the concepts of speed, magnitude of acceleration, and direction.

17. (a) Find the velocity and acceleration vectors for the hyperbola, given

$$R(t) = (b \sec t)\,\vec{i} + (b \tan t)\,\vec{j}.$$

(b) Find the velocity and acceleration vectors and their magnitude, given:

$$R(t) = e^{t}\,\vec{i} + e^{\frac{t}{2}}\,\vec{j}$$

$$R(t) = (\sin t)\,\vec{i} + (\cos 2t)\,\vec{j}.$$

(c) A particle moves in the xy-plane so that at any time $t \geq 0$ its position (x,y) is given by

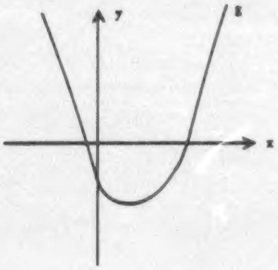
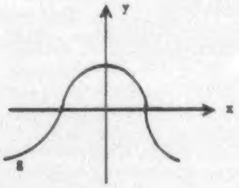
$$x = e^{t} + e^{-t},\ y = e^{t} - e^{-t}.$$

(1) Find the velocity vector for any $t \geq 0$.

(2) Find $\lim\limits_{t \to \infty} \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

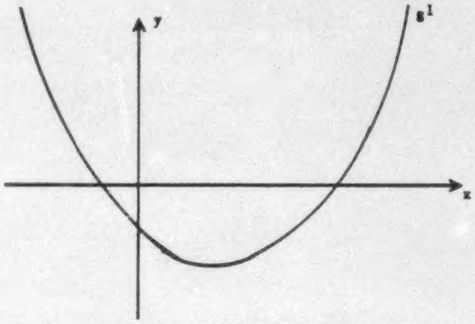
(3) Find an equation in terms of x and y for the path of the particle; indicate domain and range.

(4) Sketch the graph of (3) and show the velocity vector for $t = 0$.

C. INTEGRAL CALCULUS *BC ONLY	
1. Recognizing the behavior of the graph of $f(x)$ and $f(x) + k$, k any real number.	1. (a) Think of the graphs of $y = 3x - 7$, $y = 3x + 2$, $y = 3x + 5$; find the slope for each function. (b) Sketch the graph of each function; express y' of each function and evaluate for each function y' at $x_1 = 1$, $x_2 = -2$, $x_3 = 2$; $y = x^3 - 4x$; $y = x^3 - 4x + 3$; $y = x^3 - 4x + 5$.
2. Recognizing the function (within reason), given the derivative; recognizing that a function has one derivative, but the antiderivative is not unique.	2. (a) Using what the questions in (1) above suggest, write for each of the following two functions, whose derivative is: $f'(x) = -3$; $f'(x) = 4x^3 - 2x$; $f'(x) = \frac{1}{x} + e^x$, $x \neq 0$. (b) Estimate the slope of the given graphs at enough points, plot these points to sketch the graph of g'.  

CALCULUS C

(c) Given the graph of g', a quadratic; sketch the graph of g. What is the degree of g'? Of g?



3. Recognizing that if F has a derivative f, F is called an antiderivative of f; finding an antiderivative by "inspection"; by the rules; finding the antiderivative given initial conditions.

3. (a) For each of the following, give an antiderivative for

sin x ______

cos x ______

$3x^2 + 2x$ ______

sec x tan x ______

(b) If the graph of $y = f(x)$ passes through the point (-2,2), and the slope at any point is $3x^2 + 4x + 3$, write the equation of $f(x)$.

(c) A particle P moves along a line in such a way that its velocity at time t is given by $v = 4t^2 + 2t - 3$; when $t = 1$, P is at point 3; where is P when $t = 6$?

4. Understanding that antidifferentiation is called integration and that $\int f(x)dx$ denotes a family of functions; recognizing and using accurately the terms integral sign, integrand, constant of integration, indefinite integral; checking integration by differentiation and recognizing that they are inverse operations.

4. (a) Evaluate each of the following indefinite integrals:

$\int(3x-2)dx$; $\int\left(\frac{3}{\sqrt{x}} + 2\sqrt[3]{x}\right)dx$;

$\int(5x-x^2+2)dx$; $\int\frac{1}{x}dx$, $x > 0$.

CALCULUS C

	(b) Complete each of the following: $\int du =$; $\int a\,du =$; $\int (du + dv) =$; $\int u^n du =$, $n \neq -1$; $\int \frac{1}{u} du = u > 0$. (c) Indicate in each of the following u and du; rewrite the integral in the form $\int u^n du$ and evaluate: $\int (3x-5)^3 dx$; $\int x\sqrt{(3x^2+5)^3}\,dx$; $\int \cos^2 y \sin y\,dy$; $\int \frac{dx}{2x-1}$. (d) A particle moves on the x-axis so that its acceleration at any time $t > 0$ is given by $a = \frac{1}{8}t - \frac{1}{t^2}$. When $t = 1$, $v = \frac{9}{16}$, and $s = \frac{25}{48}$, find the velocity v in terms of t; find the distance s from the origin at time $t = 2$.
5. Recognizing expressions like $\frac{dy}{dx} = f(x)$, as differential equations and solving, given initial conditions; applying solutions of $\frac{dy}{dx} = ky$ to problems of exponential growth and decay.	5. (a) Find the function that satisfies the equation $\frac{dy}{dx} = \frac{x^2+1}{x^2}$, if $y = 1$, when $x = 1$. (b) Find the velocity v and the position s as functions of t, given that the acceleration $a = 6t$, and when $t = 1$, $v = 4$, and $s = 5$. (c) Use the law of exponential growth, $y = Ce^{kt}$, to find the growth function whose graph passes through $(0, \frac{1}{2})$, and $(5,5)$. (d) If $y = Ce^{kx}$, find $\frac{dy}{dx}$; find $y = f(x)$, if $\frac{dy}{dx} = 5y$

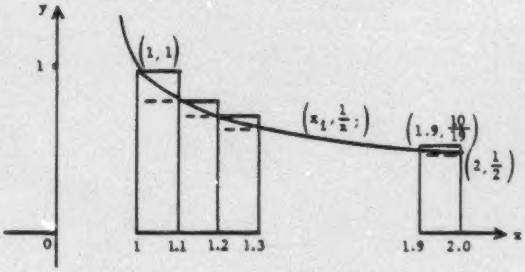
CALCULUS C

6. Applying techniques of integration; basic integration formulas; integration by substitution; integration by parts.	6. Evaluate the following integrals: (a) $\int(3x^2-2x+5)dx$; $\int(x^2-\sqrt[3]{x})dx$; $\int\frac{2x-2}{x^2-2x+3}dx$; $\int 3\sin x\,dx$; $\int\tan x\,dx$. (b) $\int 3t^2(t^3-5^3)^2dt$; $\int\sec^2x\,dx$; $\int\cos^2x\sin x\,dx$; $\int\frac{(\cos 2x)dx}{\sin^2 2x}$. (c) $\int xe^x dx$; $\int x\sec^2x\,dx$; $\int\ln x\,dx$.
*7. Evaluating integrals by (repeated) integration by parts; integration by partial fractions; integration by trigonometric substitution; trigonometric integration.	7. $\int\frac{\sqrt{x-1}}{x}dx$; $\int x^2e^x dx$; $\int\sqrt{1+\sin x}\,dx$; $\int\sin^5x\cos^2x\,dx$; $\int\frac{\sqrt{25-x^2}}{x}dx$; $\int\frac{x^2-1}{x^3+x}dx$; $\int x\sin x^2dx$.
8. Understanding the concept of the definite integral as an area; limits of integration; illustrating by diagram; approximating the definite integral by rectangles; realizing that only now can areas, bounded by curves, be evaluated.	8. (a) Draw the graph of $f(x) = \frac{2}{3}x + 2$; shade the region bounded by the graphs of f, $y = 0$, $x = 0$, and $x = 5$. Compute the area of the shaded region; evaluate $\int_0^5(\frac{2}{3}x+2)dx$. (b) Repeat above for $f(x) = -\frac{1}{3}x + 4$, region bounded by graphs of f, $x = 0$, $y = 0$.

(c) Given $y = \frac{1}{4}x^2+2, \; 0 \leq x \leq 2,$

draw the graph, inscribe and circumscribe rectangles, with base = $\frac{1}{2}$; approximate area under the curve by upper sum; by lower sum; find area under the curve, using the definite integral.

(d) Approximate the area, bounded from above by $y = \frac{1}{x}$, from below by the x-axis, and by x = 1 and x = 2, by inscribed and circumscribed rectangles at x = 1, 1.1, 1.2, 1.3, . . . 2. Use a calculator or table of reciprocals to evaluate:



$$\text{Kcir.} = .1 \left(\frac{10}{10} + \frac{10}{11} + \frac{10}{12} + \ldots + \frac{10}{19}\right)$$

$$= 1\left(\frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \ldots \frac{1}{19}\right) =$$

K(insc) = ?   Find average;

evaluate $\int_1^2 \frac{dx}{x}$; use table,

compare results.

9. Stating the definition of the definite integral as the limit of a sum; recognizing the relation of continuity of f on [a,b] to the choice of c_i, $a \leq x_i < c_i < x_{i+1} \leq b$, and $f(c_i)$; recognizing that the indefinite integral denotes a family of functions whereas the definite integral denotes a number that is the limit of a sum.

9. (a) f is continuous on [a,b] and $a \leq x_i \leq c_1 < c_2 \leq x_{i+1} \leq b$; compare $\lim\limits_{x \to c_1} f(x)$ with $\lim\limits_{x \to c_2} f(x)$, if $\Delta x = x_{i+1} - x_i$ and $\Delta x \to 0$.

(b) Assume that f is continuous on [1,7] and that

$\int_1^5 f(x)dx = 3, \quad \int_2^3 f(x)dx = 1,$

$\int_3^5 f(x)dx = 1, \quad \int_3^7 f(x)dx = 6;$

Evaluate: $\int_1^3 f(x)dx; \int_5^7 f(x)dx;$

$\int_2^5 f(x)dx; \int_2^7 f(x)dx; \int_1^2 f(x)dx; \int_1^7 f(x)dx.$

(c) Assume that g is continuous on [1,9] and that

$\int_1^4 g(x)dx = 1, \int_2^4 g(x)dx = 2;$

$\int_2^6 g(x)dx = 0; \int_6^9 g(x)dx = 1;$

Evaluate; $\int_1^2 g(u)du; \int_2^9 g(v)dv;$

$\int_4^6 g(w)dw; \int_1^9 g(x)dx; \int_4^9 g(y)dy;$

$\int_1^6 g(g)dg.$

(d) Draw the graph of $y = x^2$, $0 \leq x \leq 1$; divide the interval into n subintervals and write an expression for the inscribed rectangles, S_L;

write an expression for the circumscribed rectangles S_U;

$\left(\text{use} \sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)\right);$ find

$\lim_{n\to\infty} S_L$, $\lim_{n\to\infty} S_U$; evaluate and compare $\int_0^1 x^2 dx$.

*10. Approximating areas by Upper and Lower (Riemann) sums and the Trapezoidal Rule; evaluating exact area if the antiderivative exists; recognizing an appropriate definite integral identified with an infinite sum and evaluating, if limit exists.

10. (a) Repeat 9(d) for $f(x) = x^3$, $0 \leq x \leq 2$.

(b) In which of the following does the Trapezoidal Rule over-estimate, underestimate the area under a curve on a given interval?

$y = \frac{1}{3}x^2+2$, $[0,3]$;

$y = \frac{1}{3}x+2$, $[0,3]$;

$y = x^{-2}$, $[1,2]$; $y = e^{2x}$, $[0,2]$;

give a reason for your answer and generalize.

(c) Use the Trapezoidal Rule to approximate area under the curve, using n = 4, and compare with exact value obtained by definite integral:

$y = \frac{1}{x^2}$, $[1,2]$; $y = \sin x$, $[0,\pi]$;

$y = x\sqrt{x^2+1}$, $[0,2]$.

(d) In each of the following, identify the integral which the sum approaches and evaluate, if the limit exists:

$$\lim_{n\to\infty} \frac{1^3+2^3+3^3+\ .\ .\ .\ + n^3}{n^4};$$

$$\lim_{n\to\infty} \frac{1 + \sqrt[n]{e} + \sqrt[n]{e^2} + \ .\ .\ .\ + \sqrt[n]{e^{n-1}}}{n};$$

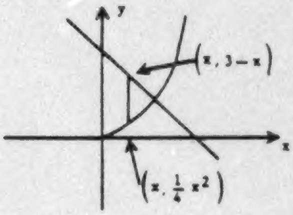
$$\lim_{n\to\infty} \frac{(1+\frac{3}{n})^2 + (1+\frac{6}{n})^2 + \ldots + (1+\frac{3n}{n})^2}{n}$$

CALCULUS C

11. Stating and applying the Fundamental Theorems of Calculus: (a) $\int_a^b f(x)dx=F(b)-F(a)$; (b) $\frac{d}{dx}\int_a^x f(t)dt=f(x)$; where $F'(x)=f(x)$ and $x\varepsilon(a, b)$. Recognizing that the precise relationship between antiderivative and integrals is called the Fundamental Theorem of Calculus; understanding that it enables us to evaluate the definite integral without using the limit of a sum, provided we can find an antiderivative for f; following through the derivation of the Fundamental Theorems; using notation accurately.	11. (a) Evaluate the definite integrals: $\int_0^2 t^3dt$; $\int_0^1 x^4dx$; $\int_1^4 \frac{ds}{\sqrt{s}}$; $\int_0^{\frac{\pi}{3}} \sec x \tan x\, dx$; $\int_0^{\pi} 3\sin x \cos^2 x\, dx$; $\int_0^{\frac{\pi}{2}} \sin x\, dx$; $\int_0^3 e^x dx$. (b) In each of the following, find $F'(x)$, given: $F(x) = \int_1^x \frac{dt}{t}$; $F(x) = \int_0^x \frac{dt}{1+t^2}$; $F(x) = \int_x^1 t^2dt$.
*12. Using Fundamental Theorem applied to $u = f(x)$ as limit of integration.	12. Differentiate with respect to x: $\int_0^{x^2-x} \tan t\, dt$; $\int_1^{x^2} \sqrt{t^4+1}\, dt$; $\int_0^{\sin x} \frac{dt}{2+t}$; $\int_3^{\sqrt{x}} (t^3+1)^{\frac{1}{2}}dt$; $\int_{\sin x}^{\cos x} \sec t\, dt$.
13. Applying integration to finding the average value (mean) of a function over an interval; computing areas between curves and volumes of a solid of revolution by different methods; choosing the appropriate method, not only to save work, but to be able to complete the evaluation; that is, writing an integral for which one can find an antiderivative.	13. (a) In each of the following, find the average value of the function on the given interval; sketch the graph of f and the rectangle whose altitude illustrates the average ordinate: $f(x) = \cos x$, $[0,\frac{\pi}{2}]$; $f(x) = x^3-1$, $[1,3]$; $f(x) = x^2$ and the interval is $[1,4]$; find the mean value of $(f-1)$; $(f-4)$; f^2.

(b) Find the area between the graphs of $y = 3 - x$ and $y = \frac{1}{4}x^2$, for $x \geq 0$.

Draw a diagram, write an expression for ΔA and A; evaluate $\Delta A =$, $A =$



Repeat for:

$$\begin{cases} f(x) = x^2 - 1; \\ g(x) = 2 - 2x^2 \end{cases}$$

$$\begin{cases} f(x) = \sqrt{x} + 2,\ x \geq 0 \\ g(x) = \frac{1}{16}\ x^2; \end{cases}$$

$$\begin{cases} f(x) = x^4 - 4x^2; \\ g(x) = 4x^2. \end{cases}$$

(c) Find the volume generated when the region bounded by the graph of $y = x^{\frac{3}{2}},\ 0 \leq x \leq 4$, is revolved about the x-axis; about the y-axis. Repeat for the region bounded by graphs $y = \sqrt{16-x^2},\ x = -2,\ x = 2,\ y = 0$.

(d) Find the volume when the region bounded by $y = x^2$ and $4x - x^2$ revolves about the y-axis; about the line $x = 4$; about the x-axis.

(e) A football has a volume approximately the same as that generated by rotating the ellipse's interior, $b^2x^2+a^2y^2=a^2b^2$, a, b constants, about the x-axis. Find the volume.

CALCULUS C

*14. Finding areas of regions bounded by polar curves; length of a path traveled by a particle; length of a curve; using symmetry to simplify the work involved.	14. (a) Find the total length of the four-cusped hypocycloid defined by the coordinates of P(x,y): $x = a \cos^3\theta,\ y = a \sin^3\theta.$ (b) Find the distance traveled between $t = 0$ and $t = \frac{\pi}{2}$ by a particle P(x,y) whose position at time t is given by $x = a \cos t + a t \sin t,$ $y = a \sin t - a t \cos t,$ where a is a positive constant. (c) Find the area of one leaf of a four-leafed rose, given its equation $r = a \sin 2\theta$. (d) Find the area outside the circle, $r = 2a \cos\theta$, and inside the cardioid, $r = a(1+\cos\theta)$.
*15. Finding volumes of solids with known cross sections; finding areas of the surface of revolution; evaluating, if the limit exists, improper integrals; recognizing the points of discontinuity of the integrand on the given interval.	15. (a) Find the volume of a solid with circular base of radius 5, if the cross sections, perpendicular to the plane of the circle, are equilateral triangles, with base perpendicular to a fixed diameter. (b) The base of a solid is bounded by $f(x) = 1 - \frac{x}{2}$, $g(x) = -1 + \frac{x}{2}$, and $x = 0$; the cross sections, perpendicular to the x-axis, are semicircles. Find V. (c) Find the area of the surface generated by rotating the curve $y = \frac{1}{2}(x^2+1),\ 0 \le x \le 1$, about the y-axis.

(d) The curve defined by

$x = t + 1,\ y = \frac{t^2}{2} + t$, from $t = 0$ to $t = 4$, is rotated about the y-axis. Find the surface area generated.

(e) Determine whether or not the integrals converge, and evaluate if the limit exists:

$$\int_0^\infty \frac{dx}{1+e^x}\ ; \qquad \int_0^\infty xe^{-x}dx\ ;$$

$$\int_0^1 x \ln x \, dx.$$

16. Learning more about integration, with suggested solutions.

16. More on integration, with solutions suggested: complete each of the following.

(a)

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{\frac{1}{2}\,dx}{\sqrt{1-\left(\frac{x-1}{2}\right)^2}} = ?$$

Let $u = \frac{x-1}{2}$ $\quad du = ?$

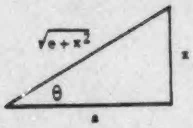
Try to compare:
$x-1 = 2\sin\theta,\ -1 < x < 3 \Longrightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

(b)

$$\int \frac{dx}{(a^2+x^2)^2} = \int \frac{a\sec^2\theta d\theta}{a^4\sec^4\theta} =$$

$\frac{1}{a}$, $\int \cos^2\theta d\theta = ?$

Let $x = a\tan\theta$
$dx = a\sec^2\theta$
$\theta = \tan^{-1}\frac{x}{a}$



(c)

$$\int\frac{dx}{(x+1)(x^2+1)} = \frac{1}{2}\int\frac{dx}{x+1} - \frac{1}{2}\int\frac{dx}{x^2+1} = ?$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$1=(A+B)x^2+(B+C)x+(A+C)$$

$$\Longrightarrow \left.\begin{array}{l} A+B = 0 \\ B+C = 0 \\ A+C = 1 \end{array}\right\} \Longrightarrow \begin{array}{l} A-C = 0 \\ A+C = 1 \end{array}$$

$$\therefore A = \tfrac{1}{2},\ B = -\tfrac{1}{2},\ C = \tfrac{1}{2}$$

(d)

$$\int\sin^6 x\,dx = \tfrac{1}{8}\int(1-\cos 2x)^3 dx$$

$$\tfrac{1}{8}\int(1-3\cos 2x+3\cos^2 2x-\cos^3 2x)dx =$$

since $\sin^2 x = \tfrac{1}{2}(1-\cos 2x)$

$$= \tfrac{1}{8}\int(1-3\cos 2x+\tfrac{3}{2}(1+\cos 4x)-(1-\sin^2 2x)\cos 2x)dx = ?$$

since $\cos^2 x = \tfrac{1}{2}(1+\cos 2x)$.

(e)

$$\int_0^1 \frac{dx}{4-x^2} = \int_0^{\frac{\pi}{6}} \frac{2\cos\theta\, d\theta}{4\cos^2\theta} = \frac{1}{2}\int_0^{\frac{\pi}{6}} (\quad) =$$

$x = 2\sin\theta$

$x = 0 \Longrightarrow \theta = 0$
$x = 1 \Longrightarrow \theta = \frac{\pi}{6}$

D. SEQUENCES AND SERIES

BC Calculus Only

1. Recalling the notion of a sequence as a function whose domain is the set of positive integers; using the theorems of limits for sums, reciprocals (nonzero terms), quotients (nonzero denominator); determining whether a sequence is divergent or convergent, and, if convergent, finding the limit.	1. (a) Plot, in the coordinate plane, the first five terms of the sequence defined by a_n; tell whether or not it converges and if it does, give the limit: $a_n = n+1$; $a_n = \frac{1}{n}$; $a_n = (-1)^n\frac{1}{n}$; $a_n = 1 - \frac{1}{n}$; $a_n = \cos(2n-1)\frac{\pi}{2}$; $a_n = n\frac{\pi}{4}$. (b) Write the first three terms and determine the convergence or divergence of the sequences defined by a_n; give the limit when it exists: $a_n = \frac{n^3-n+1}{n^3+n-1}$; $a_n = \frac{n+1}{n^2+1}$; $a_n = \frac{n}{\log n}$; $a_n = \sin n$; $a_n = n(e^{\frac{1}{n}} - 1)$; (hint: let $n = \frac{1}{t}$).
2. Following derivation and applying theorem: If $\lim\limits_{x\to\infty} f(x) = L$, and if $f(n) = a_n$, then $\lim\limits_{n\to\infty} a_n = L$; using the "Sandwich" or "Squeeze" Theorem to find limit of sequences; deriving and developing facility in applying limits that occur frequently; using l'Hopital's Rule to evaluate limits of sequences.	2. (a) Find $\lim\limits_{n\to\infty} \frac{\ln n}{n}$; $\lim\limits_{n\to\infty} e^{-n}$; $\lim\limits_{n\to\infty} \frac{1}{n!}$; $\lim\limits_{n\to\infty}(\sin^2\frac{1}{n}+\cos^2\frac{1}{n})$. (b) In the same set of axes, plot the sequences $\left\{\frac{1}{2^n}\right\}$, $\left\{\frac{-1}{2^n}\right\}$, and $\left\{\frac{(-1)^n}{n!}\right\}$; find the limit of each; find N, such that for $n > N$ $\frac{-1}{2^n} \leq \frac{(-1)^n}{n!} \leq \frac{1}{2^n}$; repeat for: $\{-e^{-n}\}$; $\{\cos n(e^{-n}\}$; $\{e^{-n}\}$.

	(c) Prove the following limits: $\lim_{n\to\infty} \frac{\ln n}{n} = 0$; $\lim_{n\to\infty} \sqrt[n]{n} = 1$; $\lim_{n\to\infty} x^{\frac{1}{n}} = 1$, $x > 0$; $\lim_{n\to\infty} x^n = 0$, for $\lvert x\rvert < 1$; find $\varepsilon > 0$, and N, such that for $n \geq N$, $\lvert x^n\rvert < \varepsilon$.
3. Recognizing a series as a special sequence; writing first few partial sums to get the pattern of the series; recognizing the finite and infinite geometric series; expressing sum, finding any S_i; recognizing and being able to compute the rational equivalent of a repeating decimal, which represents an infinite geometric series.	3. (a) Find the sum of the first ten terms of: $2 + 4 + 8 +, \dots;$ $2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots;$ (b) Write the n^{th} term of $S = 5 - \frac{5}{2} + \frac{5}{4} - \dots;$ write $\frac{S}{2}$, add it to S, and evaluate. (c) Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \dots;$ $1 + .03 + .0003 + .0000003 + \dots;$ write, in simplified form, the rational equivalent of $1.136\overline{36}$.
4. Recognizing that if $\lim_{n\to\infty} a_n \neq 0$, or $\lim_{n\to\infty} a_n$ fails to exist, then $\sum_{n=1}^{\infty} a_n$ diverges.	4. (a) Examine $\{a_n\}$ and $\left\{\frac{1}{a_n}\right\}$; If $\sum_{n=1}^{\infty} a_n$ converges, does $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converge? Explain.

(b) Again, examine

$$\lim_{n \to \infty} \sum_{j=1}^{n} \frac{1}{j} \;;$$

Draw the graph of $f(x) = \frac{1}{x}$, $x > 0$; illustrate the infinite sum by comparing it to the area under the curve; $n \in N$, $x \in R$; is $f(n) = a_n$?

$$\sum_{n \to \infty} \frac{1}{x} \;?$$

What can you conclude about

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots ,$$

a special series, called the harmonic series?

(c) Write the first four partial sums; determine whether the series converges or diverges; justify your answer:

$$\sum_{n=1}^{\infty} (-1)^{n-1}; \quad \sum_{n=1}^{\infty} \frac{n}{1+n}; \quad \sum_{n=1}^{\infty} \frac{n+1}{n}$$

5. Recognizing and applying the theorems for sums, quotients, and multiples of convergent series; examining a given series and, if possible, rewriting in a simpler form before trying to evaluate the sum; working carefully with "Σ" notation and indexing of terms.

5.(a) Rewrite each sum in a closed form and give the value, if the series converges:

$\ln 2 + \ln 3 + \ln 4 + \ldots \ln n$;

$$\ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} + \ldots + \ln \frac{n}{n+1} \;;$$

$$\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots + \frac{1}{(n+1)(n+2)} \;;$$

$$\left(\text{Note} \quad \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}\right);$$

$$\sum_{n=1}^{\infty} \frac{3}{(3n-2)(3n+1)}$$

(b) If:

$$\lim_{n\to\infty}\sum_{k=1}^{n} a_k = L, \text{ express } \lim_{n\to\infty}\sum_{k=21}^{n} a_k\ ;$$

find the difference of the sums:

$$\sum_{n=1}^{\infty}\frac{3^{n-1}-2^{n-1}}{6^{n}} \quad\text{and}\quad \sum_{n=2}^{\infty}\frac{3^{n-2}-2^{n-2}}{6^{n-1}}\,.$$

6. Testing convergence by the following methods: checking nth term; comparison test; limit comparison test; ratio test; distinguishing between increasing and decreasing partial sums and upper and lower bound; recognizing the divergence of harmonic series.

6. (a) Illustrate by plotting the graph of a series with increasing partial sums and $S_n \leq M$; $S_n > M$; give an example for each series above.

(b) Why does

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots \text{ converge?}$$

What is an upper bound for

$$S_n=\sum_{n=0}^{\infty}\frac{1}{n!}, \text{ if } S_n < 1+\sum_{n=0}^{\infty}\frac{1}{2^n}\ ?$$

(c) In each case determine whether the series converges or diverges, and give a reason for your answer:

$$\sum_{n=1}^{\infty}\frac{n}{n+10}\ ;\quad \sum_{n=1}^{\infty}(2)(\tfrac{1}{3})^{n}\ ;\quad \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\ ;$$

$$\sum_{n=1}^{\infty}\frac{n+1}{n^{3}}\ ;\quad \sum_{n=1}^{\infty}\frac{1}{1+e^{n}}\ ;\quad \sum_{n=1}^{\infty}\frac{\sin^{2}n}{2^{n}}\,.$$

$$\sum_{n=1}^{\infty}\frac{n^{3}}{3^{n}}\ ;\quad \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\ ;\quad \sum_{n=1}^{\infty}\frac{10n+3}{n2^{n}}\ ;$$

$$\sum_{n=1}^{\infty}\frac{2^{n}}{n^{2}}\ ;\quad \text{what is } \lim_{n\to\infty} a_n\ ?$$

CALCULUS D

7. Stating, giving the hypothesis accurately, test of convergence or divergence by integral test; writing the function that matches the positive $\sum a_n$, with $f(n) = a_n$; recognizing when a_n is defined by a formula and $f(x)$ can be defined by the same formula; using integral test to approximate sums; determining convergence of p-series.	7. (a) Test each of the following series for convergence or divergence and give a reason for each answer: $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$; $\sum_{n=3}^{\infty} \frac{1}{n\ln^2 n}$; $\sum_{n=1}^{\infty} \frac{n}{e^n}$; $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \ . \ . \ . \ .$; $\sum_{k=1}^{\infty} \frac{1}{\sqrt{3k+5}}$; $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$. (b) The n^{th} term of a series is $a_n = \frac{x^{2n-1}}{2n-1}$, x positive; for what values of x does the series converge? Diverge? (c) If the terms of a series are negative, can the tests for convergence or divergence of series with positive terms be applied? Explain.
8. Defining absolute convergence; stating and applying absolute convergence theorem; considering converse of theorem.	8. (a) Write the first five terms of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ and the corresponding series of absolute values; determine whether the series converges absolutely and why; repeat for $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$; $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n+1}$; $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n} \right)$ (b) Given: $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ and $\sum_{n=0}^{\infty} \frac{1}{2^n}$; do both series converge? If your answer is yes, do they converge to the same number?

9. Recognizing alternating series; stating conditions for strictly alternating series for convergence; examining the three conditions: can any one be omitted? Why or why not? Estimating the error incurred when approximating the sum; recognizing absolute and conditional convergence.

9. (a) If an alternating sum converges to L and an approximation of the sum of n terms is S_n, show, for $\sum_{i=1}^{\infty} (-1)a_i$, what the maximum error would be.

(b) Does $\sum_{k=1}^{\infty} (-1)\frac{1}{k}$ converge? Prove your answer. Compare the series to $\sum_{i=1}^{\infty} \frac{1}{n}$; draw a conclusion regarding absolute convergence.

(c) Prove whether or not $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{2^{n-1}}$ converges.

(d) An approximation for ln(1+x), for $|x| < 1$, is given by

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n+1}\frac{x^n}{n};$$

does the series converge? If so, how good is the approximation for ln(1.2), if the first three terms are used?

(e) Determine which of the following are absolutely convergent or conditionally convergent:

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+5}; \quad \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3+n}{5+n};$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}; \quad \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{10}\right)^n;$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} n^{-\frac{1}{2}}.$$

10. Defining a power series; determining interval of convergence; following through derivation of coefficients and computing them for an nth degree Taylor polynomial expansion at x=0 and x=a; noting that the terms of a power series are variable; realizing that power series tell us about the "origin" of log, trig, etc., tables; evaluating remainder; recognizing a possible change in interval of convergence when operating with power series.

10. (a) A function is represented by a power series

$$\sum_{n=0}^{\infty} a_n x^n \ ;$$

if successive approximations of f are given by one, two, three, . . . term(s), interpret graphically, what the approximations represent.

(b) Write, in terms of f and its derivatives, the first four terms of the Taylor polynomial for $f_n(x)$, at $x = 0$.

(c) Write the first five terms of the Taylor polynomial for: $f(x) = e^x$; cos x; show the computation of the coefficients.

(d) Repeat (b) above for x = a; write the Taylor polynomial for

$\sin(x + \frac{\pi}{2})$.

(e) Write the first three nonzero terms in the Taylor series expansion of cos x about

$x = \frac{\pi}{2}$; show the computation in well-organized form; give the interval of convergence, showing your method; use the first two nonzero terms to approximate

$\cos(\frac{\pi}{2} + 0.1)$, and estimate the

accuracy of the approximation, showing your work.

(f) Write the Taylor series expansion for $f(x) = \ln(1+x)$ about $x = 0$. Include an expression for the general term. Divide 1 by 1+t; write six terms of quotient and the general term. Integrate the resulting series from 0 to x, i.e.,

$$\int_0^x \frac{1}{1+t}\,dt = \int_0^x (\qquad)\,dt.$$

State the result and compare with Taylor expansion. For what values of x does the series for $\ln(1+x)$ converge? Show your work in detail.

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11. Using power series of function and deriving by substitution, or other operations, power series of related functions.

11. (a) Write the Taylor expansion for $f(x) = e^x$; show how you can obtain the series of

e; e^{x^2}; e^{x^2+1} $(= e \cdot e^{x^2})$;

(b) Repeat for cos x: Write the series for $\cos\sqrt{x}$; find the power series for $\sin^2 x$, using the identity

$$\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{\cos(2x)}{2}.$$

(c) Estimate $\int_0^1 e^{-x^2}dx$ to two decimal places by writing first the series for e^{-x^2} and integrating. Find the error if you add four terms.

E. DIFFERENTIAL EQUATIONS

CALCULUS BC Only

1. Classifying ordinary differential equations according to order and degree; recognizing that no degree is assigned if the equation is not of polynomial form, e.g., $y'' - \sin y = 0$ is of second order - no degree assigned; understanding the concept of solution, general and particular, and the meaning of initial conditions.	1. (a) Indicate the order and degree of each of the following differential equations: $y' - 2y(y')^2 + x = 0$; $(y''')^2 + 4y = 2$; $y' - 3y = e^x$; $y' - 3e^x y = 0$. (b) Show whether or not the given functions are solutions of $y' - y = 0$: $y = \sin x$; $y = e^x$; $y = e^{2x}$; $y = 0$; $y = 4e^{-x}$. (c) Show that $y = \frac{3}{2}x^2 + 2x - 1$ is a solution of $y'' - (\frac{1}{x})y' + \frac{2}{x} = 0$. (d) $y = Ce^{-2x}$ is a general solution of $y' + 2y = 0$; find the particular solution if $y = 3$, when $x = 0$ (also expressed as $y(0) = 3$), and check.
2. Solving first-order differential equations with variable separable; recognizing the differential form as well as the derivative form; performing the necessary integration.	2. Give the general solution of each of the following: $\frac{dy}{dx} = \frac{x}{y}$; $\frac{dy}{dx} = xy$; $x^2dy + y^2dx = 0$; $ydy = \sin x\,dx$; $(x^2+3)dy + x(2y-1)dx = 0$; $\frac{dy}{dx} = e^{x-y}$; $\frac{x}{y} = \ln x \frac{dx}{dy}$; $(x+1)dy = x(y^2+1)dx$.

CALCULUS E

3. Solving first-order linear differential equations that can be written in the form $\frac{dy}{dx} + P(x)y = Q(x)$; recognizing when $\frac{dx}{dy} + P(y)x = Q(y)$ is preferable; recognizing the method of integration involved.	3. Solve each of the following: $\frac{dy}{dx} + 2y = e^{-x}$; $x\,dy - (x^4+2y)dx = 0$; $y' - y = \cos x$ and $y(0) = \frac{1}{2}$; $(2xy+1)dy + (y^2+1)dx = 0$.
4. Solving differential equations that involve the substitution for $\frac{y}{x} = v$; recognizing that after substituting, the equation can be transformed into one with variable separable; solving differential equations that are exact.	4. (a) Solve each equation: $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$; $(xe^{\frac{y}{x}} + y)\,dx - x\,dy = 0$; $\frac{dy}{dx} = \frac{-x + \sqrt{x^2+y^2}}{y}$; (b) Determine whether the equation is exact and find the general solution: $(2x+y)dx + (x+2y)dy = 0$; $(2xe^y + e^x)dx + (x^2+1)e^y dy = 0$; $(e^x + \ln y + \frac{y}{x})dx + (\frac{x}{y} + \ln x + \sin y)dy = 0$.
5. Solving second-order linear differential equations with constant coefficients, using the substitution $\frac{dy}{dx} = p \Rightarrow \frac{d^2y}{dx^2} = \frac{dp}{dx}$.	5. Solve each of the following: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$; $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$; $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

6. Solving, with the aid of the characteristic equation, second-order linear differential equations, homogeneous and nonhomogeneous; writing and solving the characteristic equation and distinguishing between the three cases of roots; finding particular solutions, given initial conditions; finding general solutions to nonhomogeneous differential equations, using method of undetermined coefficients.

6. (a) Solve each of the following differential equations:

$y'' + 6y' + 12y = 0;$

$y'' + 4y' + 4y = 0$ and

$y(0) = 2,\ y'(0) = 1;$

$\frac{d^2y}{dx^2} + 3y = 1 - e^{-x}.$

(b) AP-BC 1982 #4:
A particle moves along the x-axis so that its position function $x(t)$ satisfies the differential equation

$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 6x = 0$

and has the property that at time $t = 0$, $x = 2$ and

$\frac{dx}{dt} = -9.$

(1) Write an expression for $x(t)$ in terms of t.

(2) At what times t, if any, does the particle pass through the origin?

(3) At what times t, if any, is the particle at rest?

(c) Try to solve:

$y''' - 6y'' + 11y' - 6y = 0.$

(d) Solve $y'' - 3y' + 2y = 2x;$

$y'' - 2y' = x + 2e^{x};$

$y'' - y = \sin x;$

$y'' + 4y' + 5y = x + 2.$

7. Solving problems of AP-BC examinations.

7. (a) If $\sqrt{2}\, e^{2x}$ is a particular solution of the differential equation $y'' - 4y' + py = 0$, find the value of p and the general solution of the equation.

Find the particular solution of the differential equation $y'' - 4y' + 5y = 0$ that satisfies the condition that $y = 0$ and

$y' = 1$ when $x = \frac{\pi}{2}$.

(b) Determine the general solution of the differential equation

(1) $y'' - 4y' + 4y = e^{2x}$

(2) Assume that xe^{x} is a solution of the differential equation $y'' + ay' + by = 0$. Determine the constants a and b.

(c) Given the differential equation $y'' + 4y' + 5y = 0$,

(1) find the general solution.
(2) find the solution that satisfies the conditions $y(0) = 0$ and $y'(0) = 1$.
(3) evaluate the limit of the solution determined in (2) as x increases without bound. Justify your answer.

F. CALCULUS AND CAREERS

Ivestigating one job from each of the three occupational clusters in which the skills learned in Differential and Integral Calculus can be applied.

ACKNOWLEDGEMENTS

Many individuals and resources helped make this document possible:

Boston high school mathematics teachers who compiled the course objectives and final examinations that were the main resources for our consultants

A committee of the 1981-2 Mathematics Council which drafted General Mathematics objectives: Joseph Sheppeck (Chairperson), Paul Natola, Frank Mangino, and Patricia Tremblay. Joseph LaCroix acted as an advisor, and Robert Hayden of M.I.T. served as a consultant

A committee of the 1981-2 Mathematics Council which drafted Algebra I objectives: Walter Casey (Chairperson), William Durante, Paul Foley, Eric Roy, and Krishna Rajangam. Joseph LaCroix acted as an advisor, and Robert Hayden of M.I.T. served as a consultant

The Mathematics Council, which, in a series of meetings during the 1982-3 school year, developed the policy statement included in the introduction:
William V. Durante, Boston Latin School - Chairperson
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Patricia Tremblay, Madison Park High School
Thomas Lydon, South Boston High School
Nicholas Rubino, Umana High School
Derrick Sudeall, West Roxbury High School

Margaret Kenney of Boston College, who provided early advice and support

Three consultants who synthesized voluminous material into initial drafts:
Bob Willcutt (Boston Univ.) - Algebra I, Algebra II, and Trigonometry
Rudd Crawford (Oberlin, Ohio) - Geometry
Martha Zelinka (Weston, Mass.) - Advanced Mathematics and Calculus

Thomas R. Deveney of the Testing Office for invaluable advice on the issue of Basic and General Mathematics

Curriculum outlines, objectives, and final exams from Boston high schools

Curriculum documents from numerous school systems, including Pittsburgh, Pennsylvania; Baltimore, Maryland; Braintree, Massachuetts; Foxborough, Massachusetts; Weston, Massachusetts; and Chicago, Illinois

Sourcebook of Applications of School Mathematics. National Council of Teachers of Mathematics

The Geometry Problem Solver and The Calculus Problem Solver. Research and Education Association, New York.

Advanced Placement Course Description, Calculus AB, Calculus BC The College Board, May 1983,May 1984. New York.

Mathematics textbooks from major publishing companies

For Basic and General Mathematics sources, see Acknowledgements in Elementary and Middle School Curriculum Objectives (1982)

Boston high school mathematics teachers, who supplied close to three hundred separate critiques of individual sections of this document in June, 1983; their comments and suggestions greatly improved the document, and their prioritizing suggestions formed the basis for decisions on circling priority objectives in the final document.

Three consultants who incorporated changes suggested by teachers and others:
Bob Willcutt - Algebra I, Algebra II, and Trigonometry
Bob Cornell (Exeter, New Hampshire) - Geometry
Martha Zelinka - Advanced Mathematics and Calculus

Several experts who gave the near-final drafts extremely careful and thoughtful readings and contributed invaluable suggestions:
Ransom Lynch (Exeter, New Hampshire)
Donald Duncan (Milton, Massachusetts)
Claire Cunningham (Lincoln, Massachusetts)
Jean Prendergast (University of Massachusetts, Boston)
Edwin Taylor (Massachusetts Institute of Technology)

Joyce Malyn-Smith of the Humphrey Occupational Resource Center, who contributed career education objectives for all courses

Merrill Gillaspy, staff assistant for the Curriculum Objectives Unit, who typed most of this document into our word processor, refined and improved the documents to their present form, and handled innumerable details involving all aspects of the project

Computer Mart of Waltham, which supplied the computer (a NorthStar Advantage) and the word processing software (Techwriter) which made possible the mathematical symbols and Greek letters; and Charles Dunning and Deena Montillo of Computer Mart, who supplied consultation at many stages of the project

Connections Information Systems, a word processing company in Cambridge, which handled the Basic and General Mathematics and the Geometry objectives and the paste-ups of the other documents

Toni Rosenberg and Susan Oleksiw, copyeditors from Editorial, Inc., who copyedited the final drafts

Beverly Zimmerman, Senior Specialist in the Curriculum Objectives Unit, who coordinated the efforts of the two committees which drafted objectives in General Mathematics and Algebra I, and provided invaluable editing and advice at every stage of the project

Kim Marshall, Program Director of the Curriculum Objectives Unit, was the overall coordinator of the project.